

1. What is the order of 8 in the additive group $\mathbb{Z}/20\mathbb{Z}$?

2. A subgroup of order 6 sits inside a group of order 42. What is its index?

3. What is the order of $(1\ 2\ 3)(4\ 5\ 6\ 7)$ in S_7 ?

4. How many conjugacy classes does S_4 have?
☐ 24
☐ 5
☐ 4
☐ 6
5. A group of order 12 need not have a subgroup of every order dividing 12.
☐ True ☐ False
6. With rightmost-first composition, $H = \{e, (1\ 2)\}$ and $g = (1\ 2\ 3)$ in S_3 . Which is gH ?
☐ $\{(1\ 2\ 3), (2\ 3)\}$
☐ $\{e, (1\ 2)\}$
☐ $\{(1\ 2\ 3), (1\ 3)\}$
☐ S_3
7. Every abelian subgroup is normal in every group containing it.
☐ True ☐ False
8. For $\varphi : \mathbb{Z}/6\mathbb{Z} \rightarrow \mathbb{Z}/6\mathbb{Z}$, $\varphi(x) = 2x$, how many elements does $(\mathbb{Z}/6\mathbb{Z})/\ker \varphi$ have?

9. How many 3-cycles are in A_4 ?

10. Which coset set fails to become a group under the usual coset multiplication?
☐ $(\mathbb{Z}/12\mathbb{Z})/\{0, 3, 6, 9\}$ (additive)
☐ $S_3/\{e\}$

☐ S_3/A_3

☐ $S_3/\{e, (1\,2)\}$

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1. **5**

In $\mathbb{Z}/n\mathbb{Z}$ the order of a is $n / \gcd(a, n) = 20 / \gcd(8, 20) = 20 / 4 = 5$. Check: $5 \cdot 8 = 40 \equiv 0$.

2. **7**

By Lagrange's theorem the index is $|G| / |H| = 42 / 6 = 7$, the number of cosets.

3. **12**

The cycles are disjoint, so the order is the lcm of their lengths: $\text{lcm}(3, 4) = 12$.

4. **5**

Conjugacy classes in S_n correspond to cycle types, i.e. partitions of n . The partitions of 4 are 4, 3+1, 2+2, 2+1+1 and 1+1+1+1, so there are 5 classes.

5. **True**

True. The converse of Lagrange's theorem fails: A_4 has order 12 but no subgroup of order 6.

6. **$\{(1\ 2\ 3), (1\ 3)\}$**

Multiply g by each element of H : $ge = (1\ 2\ 3)$ and $g(1\ 2) = (1\ 3)$. The other two-element set listed is Hg .

7. **False**

The subgroup $\{e, (1\ 2)\}$ is abelian but not normal in S_3 . The valid rule requires the ambient group to be abelian.

8. **3**

The kernel is $\{0, 3\}$, so the index is $6/2 = 3$. The image $\{0, 2, 4\}$ also has three elements; the six-element codomain is not the image.

9. **8**

Choose three of the four symbols in 4 ways. Each chosen triple has two cyclic orientations, giving $4 \times 2 = 8$. All 3-cycles are even.

10. **$S_3/\{e, (1\ 2)\}$**

Only $\{e, (1\ 2)\}$ in S_3 is not normal. A_3 has index 2, every subgroup of the additive abelian clock group is normal, and the identity subgroup is always normal.