Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. $y' = 2y$, $y(0) = 5$. Find $y(\ln 2)$.

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2. The general solution of $y'' - 6y' + 9y = 0$ is

   ☐ $C_1 \cos 3t + C_2 \sin 3t$

   ☐ $C_1 e^{3t} + C_2 e^{-3t}$

   ☐ $(C_1 + C_2 t)e^{3t}$

   ☐ $C_1 e^{3t} + C_2 e^{3t}$

3. $y'' + 9y = 0$, $y(0) = 0$, $y'(0) = 6$. Find $y(\pi/6)$.

   _____

4. $y' = 3y^{2/3}$, $y(0) = 0$ has exactly one solution.

   ☐ True ☐ False

5. The characteristic roots are $3 \pm 5i$. The solutions oscillate as $\cos \beta t$ and $\sin \beta t$. Find β.

   _____

6. Which constant solutions of $y\prime = y(1 - y)$ must be checked before dividing by $y(1 - y)$?

   ☐ Only y = 0

   ☐ No constant solutions

   ☐ Only y = 1

   ☐ Both y = 0 and y = 1

7. The solution of $y\prime = ty^2$, $y(0) = 1$, is $2/(2-t^2)$. Its maximal interval containing the initial point is

   ☐ $(-\infty, \sqrt{2})$

   ☐ $\mathbb{R} \setminus \{-\sqrt{2}, \sqrt{2}\}$

   ☐ $\mathbb{R}$

   ☐ $(-\sqrt{2}, \sqrt{2})$

8. For $y\prime + 2y = 4$, an integrating factor is e^{kt}. Find k.

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10. If the derivative of f with respect to y does not exist at the initial point, the initial-value problem must have more than one solution.

    ☐ True ☐ False

Answers ·

GRE Subject Test · GRE Mathematics

1. **20**

   $y = 5e^{2t}$, $e^{2\ln 2} = 4$.

2. $(C_1 + C_2t)e^{3t}$

   The characteristic equation r^2 − 6r + 9 = (r − 3)^2 = 0 has the repeated root 3, so the second solution is t e^{3t}: y = (C_1 + C_2t)e^{3t}. Writing C_1e^{3t} + C_2e^{3t} gives only one independent solution.

3. **2**

   $y = 2\sin 3t$; $\sin(\pi/2) = 1$.

4. **False**

   False. Both $y = 0$ and $y = t^3$ satisfy the equation and initial value, proving nonuniqueness. The right-hand side is not locally Lipschitz at zero, so the usual sufficient uniqueness theorem does not apply; that failed hypothesis alone would not have been a proof.

5. **5**

   Roots $\alpha \pm i\beta$ give $e^{\alpha t}\cos\beta t$ and $e^{\alpha t}\sin\beta t$.

6. **Both y = 0 and y = 1**

   For a constant solution the derivative is zero, so y(1 − y) = 0. Both 0 and 1 work in the original equation, although division by y(1 − y) would exclude them.

7. $(-\sqrt{2}, \sqrt{2})$

   The poles are ±√2. The interval must be connected and contain 0, so it is the component between them. Deleting two points from ℝ gives three separate intervals, not one IVP interval.

8. **2**

   In normal form p(t) = 2, so $\mu = \exp(\int 2\ dt) = e^{2t}$. Multiplying turns the left side into the derivative of $e^{2t}y$.

9. **1.75**

   The solution is $y = 2 - e^{-2t}$. At t = ln 2, $e^{-2\ln 2} = 1/4$, so y = 7/4 = 1.75.

10. **False**

    For $y' = |y|$, the derivative with respect to y fails at zero, but the right-hand side is Lipschitz and $y(0) = 0$ has the unique solution y = 0. The continuous-partial-derivative condition is sufficient, not necessary.