

8. C.3 · Differential equations and initial conditions

Starter Quiz ·

GRE Subject Test · GRE Mathematics

Name: _____ Score: _____

1. $y' = 2y$, $y(0) = 5$. Find $y(\ln 2)$.

2. The general solution of $y'' - 6y' + 9y = 0$ is

☐ $C_1 \cos 3t + C_2 \sin 3t$

☐ $C_1 e^{3t} + C_2 e^{-3t}$

☐ $(C_1 + C_2 t)e^{3t}$

☐ $C_1 e^{3t} + C_2 e^{3t}$

3. $y'' + 9y = 0$, $y(0) = 0$, $y'(0) = 6$. Find $y(\pi/6)$.

4. $y' = 3y^{2/3}$, $y(0) = 0$ has exactly one solution.

☐ True ☐ False

5. The characteristic roots are $3 \pm 5i$. The solutions oscillate as $\cos \beta t$ and $\sin \beta t$. Find β .

6. Which constant solutions of $y' = y(1 - y)$ must be checked before dividing by $y(1 - y)$?

☐ Only $y = 0$

☐ No constant solutions

☐ Only $y = 1$

☐ Both $y = 0$ and $y = 1$

7. The solution of $y' = ty^2$, $y(0) = 1$, is $2/(2 - t^2)$. Its maximal interval containing the initial point is

☐ $(-\infty, \sqrt{2})$

☐ $\mathbb{R} \setminus \{-\sqrt{2}, \sqrt{2}\}$

☐ $\mathbb{R}$

☐ $(-\sqrt{2}, \sqrt{2})$

8. For $y' + 2y = 4$, an integrating factor is e^{kt} . Find k .

9. For $y' + 2y = 4$, $y(0) = 1$, find $y(\ln 2)$.

10. If the derivative of f with respect to y does not exist at the initial point, the initial-value problem must have more than one solution.

☐ True ☐ False

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Answers ·

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1. **20**

$$y = 5e^{2t}, e^{2\ln 2} = 4.$$

2. $(C_1 + C_2t)e^{3t}$

The characteristic equation $r^2 - 6r + 9 = (r - 3)^2 = 0$ has the repeated root 3, so the second solution is $t e^{3t}$: $y = (C_1 + C_2t)e^{3t}$. Writing $C_1 e^{3t} + C_2 e^{3t}$ gives only one independent solution.

3. **2**

$$y = 2 \sin 3t; \sin(\pi/2) = 1.$$

4. **False**

False. Both $y = 0$ and $y = t^3$ satisfy the equation and initial value, proving nonuniqueness. The right-hand side is not locally Lipschitz at zero, so the usual sufficient uniqueness theorem does not apply; that failed hypothesis alone would not have been a proof.

5. **5**

Roots $\alpha \pm i\beta$ give $e^{\alpha t} \cos \beta t$ and $e^{\alpha t} \sin \beta t$.

6. **Both $y = 0$ and $y = 1$**

For a constant solution the derivative is zero, so $y(1 - y) = 0$. Both 0 and 1 work in the original equation, although division by $y(1 - y)$ would exclude them.

7. $(-\sqrt{2}, \sqrt{2})$

The poles are $\pm\sqrt{2}$. The interval must be connected and contain 0, so it is the component between them. Deleting two points from $\mathbb{R}$ gives three separate intervals, not one IVP interval.

8. **2**

In normal form $p(t) = 2$, so $\mu = \exp(\int 2 dt) = e^{2t}$. Multiplying turns the left side into the derivative of $e^{2t}y$.

9. **1.75**

The solution is $y = 2 - e^{-2t}$. At $t = \ln 2$, $e^{-2\ln 2} = 1/4$, so $y = 7/4 = 1.75$.

10. **False**

For $y' = |y|$, the derivative with respect to y fails at zero, but the right-hand side is Lipschitz and $y(0) = 0$ has the unique solution $y = 0$. The continuous-partial-derivative condition is sufficient, not necessary.