1. Which function is analytic on all of $\mathbb{C}$?

   ☐ $\operatorname{Re} z$

   ☐ $\bar{z}$

   ☐ $z^2 + 3z$

   ☐ $|z|^2$

2. The residue of $\dfrac{e^{3z}}{(z-2)^2}$ at $z = 2$ is

   ☐ $9e^6$

   ☐ 0

   ☐ e^6

   ☐ $3e^6$

3. For the unit circle traversed once counterclockwise, $\oint_{|z|=1} \dfrac{z^2+1}{z}\,dz = 2\pi i \cdot k$. Find k.

   _____

4. A pole outside the contour adds its residue to the integral.

   ☐ True ☐ False

5. For complex conjugation, the difference quotient at zero is 1 along real increments and -1 along imaginary increments. What follows?

   ☐ The function is analytic at zero.

   ☐ The complex derivative at zero does not exist.

   ☐ The derivative is 1.

   ☐ The derivative is 0, the average of the two values.

6. For $f(z) = |z|^2$, find the complex derivative $f'(0)$.

   _____

7. The Cauchy–Riemann equations at one point, without any further regularity hypothesis, guarantee a complex derivative there.

   ☐ True ☐ False

8. For $f(z) = (z^2 - 1)/(z - 1)$ away from 1, what kind of singularity is at 1?

   ☐ A pole of order 2

   ☐ A simple pole

   ☐ A removable singularity

9. Find the residue of $5/(z-1)^2$ at 1.

   _____

10. The circle $|z| = 2$ is traversed once clockwise. If $\oint_C dz/[z(z-3)] = 2\pi i\, k$, find k.

    _____

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1. $z^2 + 3z$

   Polynomials in z satisfy Cauchy–Riemann everywhere.

2. $3e^6$

   $g = e^{3z}$, $g'(2) = 3e^6$.

3. **1**

   $(z^2 + 1)/z = z + 1/z$, so the residue at the only pole $z = 0$ (inside $|z| = 1$) is 1. The integral is $2\pi i \cdot 1$, giving $k = 1$.

4. **False**

   Only enclosed singularities count.

5. **The complex derivative at zero does not exist.**

   A complex derivative must be one common limit for every direction. Two different directional limits rule it out; averaging them has no role in the definition.

6. **0**

   The difference quotient at zero is $|h|^2/h = \bar{h}$, which tends to 0 for every complex approach. The derivative exists only at zero; this does not make the function analytic on a neighbourhood.

7. **False**

   For $F(0) = 0$ and $F(z) = \bar{z}^2/z$ away from zero, the equations hold at zero. But $F(h)/h$ is 1 along either axis and -1 along the diagonal, so the derivative does not exist.

8. **A removable singularity**

   Cancellation gives $z + 1$ for $z \neq 1$. Defining $f(1) = 2$ makes the function analytic there; there is no nonzero principal part.

9. **0**

   The Laurent series contains the term $5(z - 1)^{-2}$ and no power-minus-one term. Its residue is 0, although the pole order is 2.

10. **0.3333333333333333**

    Only the pole at zero is enclosed, with residue $-1/3$. Clockwise orientation gives $-2\pi i$ times that residue, so $k = +1/3$.

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