1. What is the supremum of $\{1 - 1/n : n \geq 1\}$?

   _____

2. Which metric space (usual distance) is complete?

   ☐ $\mathbb{Q}$

   ☐ $[0, 1]$

   ☐ $[0, 1] \cap \mathbb{Q}$

   ☐ $(0, 1)$

3. If, for each $\varepsilon > 0$, one $\delta > 0$ controls all domain pairs, the function is _____ continuous.

   _____

4. Which subset of $\mathbb{R}$ is compact?

   ☐ $[0, \infty)$

   ☐ $\{0\} \cup \{1/n : n \geq 1\}$

   ☐ $\{1/n : n \geq 1\}$

   ☐ $(0, 1)$

5. Every continuous function on $(0, 1)$ is bounded.

   ☐ True ☐ False

6. The identity map $f(x) = x$ on $\mathbb{R}$ is uniformly continuous even though its range is unbounded.

   ☐ True ☐ False

7. For $f(x) = 1/x$, $x_n = 1/(n + 1)$ and $y_n = 1/(n + 2)$, what is $|f(x_2) - f(y_2)|$?

   _____

8. Why does $K = [0, 1] \cap \mathbb{Q}$ not contradict Heine–Borel in Euclidean space?

   ☐ K is unbounded in Q

   ☐ K is closed in Q but not closed in R

   ☐ All rational sets are open in R

   ☐ Heine–Borel requires K to be finite

9. A continuous real function on $[0, 1]$ satisfies $f(0) = -2$ and $f(1) = 3$. Which conclusion is guaranteed?

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   ☐ It is differentiable everywhere

   ☐ It is increasing

   ☐ It has exactly one zero

10. With usual distance, which space is complete and connected but not compact?

    ☐ $[0, 1]$

    ☐ $\mathbb{R}$

    ☐ $\mathbb{Q}$

    ☐ $(0, 1)$

1. **1**

   The values increase to 1 without reaching it.

2. $[0, 1]$

   The closed interval is a closed subset of the complete real line. The other spaces omit limits of some Cauchy sequences under their stated usual distances.

3. **uniformly**

   Uniformly continuous. Delta may depend on the chosen output tolerance but not on the pair of domain points.

4. $\{0\} \cup \{1/n : n \geq 1\}$

   As a subset of the real line with usual distance, the set containing zero and all 1/n is closed and bounded. Including its only accumulation point is essential.

5. **False**

   False: 1/x is continuous on the open unit interval and becomes arbitrarily large near zero. Failure of a theorem's hypotheses alone would not prove unboundedness; this explicit counterexample does.

6. **True**

   True. For every $\varepsilon > 0$, choosing $\delta = \varepsilon$ controls all pairs because $|f(x) - f(y)| = |x - y|$.

7. **1**

   The two inputs are 1/3 and 1/4; the outputs are 3 and 4. Their distance is 1.

8. **K is closed in Q but not closed in R**

   Heine–Borel for subsets of the real line requires closedness in the real line. Irrational limits such as $\sqrt{2}/2$ show that K is not closed there.

9. **It has at least one zero in (0,1)**

   The intermediate value theorem gives at least one zero between the endpoint values. It does not guarantee uniqueness, monotonicity or differentiability.

10. $\mathbb{R}$

    The real line is complete and connected. Its unboundedness prevents compactness. The closed unit interval is compact; the open interval and the rationals are not complete.

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