

1. What is the supremum of $\{1 - 1/n : n \geq 1\}$?

2. Which metric space (usual distance) is complete?

☐ $\mathbb{Q}$

☐ $[0, 1]$

☐ $[0, 1] \cap \mathbb{Q}$

☐ $(0, 1)$

3. If, for each $\varepsilon > 0$, one $\delta > 0$ controls all domain pairs, the function is _____ continuous.

4. Which subset of $\mathbb{R}$ is compact?

☐ $[0, \infty)$

☐ $\{0\} \cup \{1/n : n \geq 1\}$

☐ $\{1/n : n \geq 1\}$

☐ $(0, 1)$

5. Every continuous function on $(0, 1)$ is bounded.

☐ True ☐ False

6. The identity map $f(x) = x$ on $\mathbb{R}$ is uniformly continuous even though its range is unbounded.

☐ True ☐ False

7. For $f(x) = 1/x$, $x_n = 1/(n+1)$ and $y_n = 1/(n+2)$, what is $|f(x_2) - f(y_2)|$?

8. Why does $K = [0, 1] \cap \mathbb{Q}$ not contradict Heine–Borel in Euclidean space?

☐ K is unbounded in $\mathbb{Q}$

☐ K is closed in $\mathbb{Q}$ but not closed in $\mathbb{R}$

☐ All rational sets are open in $\mathbb{R}$

☐ Heine–Borel requires K to be finite

9. A continuous real function on $[0, 1]$ satisfies $f(0) = -2$ and $f(1) = 3$. Which conclusion is guaranteed?

- ☐ It has at least one zero in $(0,1)$
- ☐ It is differentiable everywhere
- ☐ It is increasing
- ☐ It has exactly one zero

10. With usual distance, which space is complete and connected but not compact?

- ☐ $[0, 1]$
- ☐ $\mathbb{R}$
- ☐ $\mathbb{Q}$
- ☐ $(0, 1)$

1. **1**

The values increase to 1 without reaching it.

2. **$[0, 1]$**

The closed interval is a closed subset of the complete real line. The other spaces omit limits of some Cauchy sequences under their stated usual distances.

3. **uniformly**

Uniformly continuous. Delta may depend on the chosen output tolerance but not on the pair of domain points.

4. **$\{0\} \cup \{1/n : n \geq 1\}$**

As a subset of the real line with usual distance, the set containing zero and all $1/n$ is closed and bounded. Including its only accumulation point is essential.

5. **False**

False: $1/x$ is continuous on the open unit interval and becomes arbitrarily large near zero. Failure of a theorem's hypotheses alone would not prove unboundedness; this explicit counterexample does.

6. **True**

True. For every $\varepsilon > 0$, choosing $\delta = \varepsilon$ controls all pairs because $|f(x) - f(y)| = |x - y|$.

7. **1**

The two inputs are $1/3$ and $1/4$; the outputs are 3 and 4. Their distance is 1.

8. **K is closed in $\mathbb{Q}$ but not closed in $\mathbb{R}$**

Heine–Borel for subsets of the real line requires closedness in the real line. Irrational limits such as $\sqrt{2}/2$ show that K is not closed there.

9. **It has at least one zero in $(0,1)$**

The intermediate value theorem gives at least one zero between the endpoint values. It does not guarantee uniqueness, monotonicity or differentiability.

10. **$\mathbb{R}$**

The real line is complete and connected. Its unboundedness prevents compactness. The closed unit interval is compact; the open interval and the rationals are not complete.