1. A 4×5 matrix has rank 3. What is the dimension of its null space?

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2. Find $\det \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}$.

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3. If A is square and $\det A = 0$, then

   ☐ A has no eigenvalues

   ☐ some entry of A is zero

   ☐ $A\mathbf{x} = \mathbf{0}$ has a nonzero solution

   ☐ A is the zero matrix

4. Which matrix is NOT diagonalisable over the reals?

   ☐ $\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$

   ☐ $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

   ☐ $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

   ☐ $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

5. The dimension of an eigenspace always equals the algebraic multiplicity of its eigenvalue.

   ☐ True ☐ False

6. For $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ and $\mathbf{b} = (3, 7)$, how many real solutions does $A\mathbf{x} = \mathbf{b}$ have?

   ☐ Exactly one

   ☐ Infinitely many

   ☐ Exactly two

   ☐ None

7. $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ row-reduces to $R = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$. Which is a basis of the original column space $\operatorname{Col} A$?

   ☐ $\{(-2, 1)\}$

   ☐ $\{(1, 0)\}$

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   ☐ $\{(1, 2), (2, 4)\}$

8. $\det A = 6$. Swap two rows, then multiply one row of the resulting matrix by 3. What is the new determinant?

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9. Over which fields is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ diagonalisable?

   ☐ Over $\mathbb{R}$ and $\mathbb{C}$

   ☐ Over $\mathbb{C}$ but not $\mathbb{R}$

   ☐ Over $\mathbb{R}$ but not $\mathbb{C}$

   ☐ Over neither field

10. Let $S = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{v} = (1, 1)$. Find c such that $S^3\mathbf{v} = c\mathbf{v}$.

    _____

1. **2**

   Rank–nullity: rank + nullity = number of columns. With 5 columns and rank 3, the null space has dimension 5 − 3 = 2.

2. **6**

   For a 2 × 2 matrix, det = ad − bc = 2 · 5 − 1 · 4 = 10 − 4 = 6.

3. **$A\mathbf{x} = \mathbf{0}$ has a nonzero solution**

   Zero determinant means a nontrivial kernel.

4. $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

   Eigenvalue 2 twice with a one-dimensional eigenspace.

5. **False**

   False. Geometric multiplicity (eigenspace dimension) is at least 1 and at most the algebraic multiplicity, and can be smaller: the matrix [[1, 1], [0, 1]] has eigenvalue 1 twice but a one-dimensional eigenspace.

6. **None**

   Subtract twice the first augmented row from the second: the last row is [0, 0 | 1], meaning 0 = 1. A free variable in the coefficient matrix does not cure inconsistency.

7. $\{(1, 2)\}$

   Use the pivot position to select column 1 of A, not R. Both original columns lie on the line through (1,2), and the second is twice the first. The two-column list is dependent.

8. **-18**

   The swap contributes −1 and the row scaling contributes 3: $(-1) \times 3 \times 6 = -18$. This scales one row, not the whole matrix.

9. **Over $\mathbb{C}$ but not $\mathbb{R}$**

   The characteristic polynomial is $\lambda^2 + 1$. It has no real roots, but has two distinct roots i and −i over $\mathbb{C}$, giving a complex eigenvector basis.

10. **27**

    S v = 3 v. Apply S three times: S^3 v = 3^3 v = 27 v. There is no need to multiply out the full matrix cube.