

1. A 4×5 matrix has rank 3. What is the dimension of its null space?

2. Find $\det \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}$.

3. If A is square and $\det A = 0$, then

- ☐ A has no eigenvalues
☐ some entry of A is zero
☐ $A\mathbf{x} = \mathbf{0}$ has a nonzero solution
☐ A is the zero matrix

4. Which matrix is NOT diagonalisable over the reals?

- ☐ $\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$
☐ $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$
☐ $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$
☐ $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

5. The dimension of an eigenspace always equals the algebraic multiplicity of its eigenvalue.

- ☐ True ☐ False

6. For $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ and $\mathbf{b} = (3, 7)$, how many real solutions does $A\mathbf{x} = \mathbf{b}$ have?

- ☐ Exactly one
☐ Infinitely many
☐ Exactly two
☐ None

7. $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ row-reduces to $R = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$. Which is a basis of the original column space $\text{Col } A$?

- ☐ $\{(-2, 1)\}$
☐ $\{(1, 0)\}$

☐ $\{(1, 2)\}$

☐ $\{(1, 2), (2, 4)\}$

8. $\det A = 6$. Swap two rows, then multiply one row of the resulting matrix by 3. What is the new determinant?

9. Over which fields is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ diagonalisable?

☐ Over $\mathbb{R}$ and $\mathbb{C}$

☐ Over $\mathbb{C}$ but not $\mathbb{R}$

☐ Over $\mathbb{R}$ but not $\mathbb{C}$

☐ Over neither field

10. Let $S = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{v} = (1, 1)$. Find c such that $S^3\mathbf{v} = c\mathbf{v}$.

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Rank-nullity: rank + nullity = number of columns. With 5 columns and rank 3, the null space has dimension $5 - 3 = 2$.

2. **6**

For a 2×2 matrix, $\det = ad - bc = 2 \cdot 5 - 1 \cdot 4 = 10 - 4 = 6$.

3. **$A\mathbf{x} = \mathbf{0}$ has a nonzero solution**

Zero determinant means a nontrivial kernel.

4. $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$

Eigenvalue 2 twice with a one-dimensional eigenspace.

5. **False**

False. Geometric multiplicity (eigenspace dimension) is at least 1 and at most the algebraic multiplicity, and can be smaller: the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ has eigenvalue 1 twice but a one-dimensional eigenspace.

6. **None**

Subtract twice the first augmented row from the second: the last row is $[0, 0 \mid 1]$, meaning $0 = 1$. A free variable in the coefficient matrix does not cure inconsistency.

7. **$\{(1, 2)\}$**

Use the pivot position to select column 1 of A, not R. Both original columns lie on the line through (1,2), and the second is twice the first. The two-column list is dependent.

8. **-18**

The swap contributes -1 and the row scaling contributes 3: $(-1) \times 3 \times 6 = -18$. This scales one row, not the whole matrix.

9. **Over $\mathbb{C}$ but not $\mathbb{R}$**

The characteristic polynomial is $\lambda^2 + 1$. It has no real roots, but has two distinct roots i and $-i$ over $\mathbb{C}$, giving a complex eigenvector basis.

10. **27**

$S\mathbf{v} = 3\mathbf{v}$. Apply S three times: $S^3\mathbf{v} = 3^3\mathbf{v} = 27\mathbf{v}$. There is no need to multiply out the full matrix cube.