2. C.2 · Multivariable calculus and vector analysis

Starter Quiz ·

GRE Subject Test · GRE Mathematics Name: _____ Score: _____

1. $\dfrac{\partial}{\partial x}(x^2 y)$ equals

   ☐ $2x$

   ☐ $2xy + x^2$

   ☐ x^2

   ☐ $2xy$

2. For $f = x^2 + y^2$, find the directional derivative at $(1, 2)$ in the direction of $(3, 4)$.

   _____

3. If $f_{xy} = 0$ everywhere on $\mathbb{R}^2$, the graph of f is a plane.

   ☐ True ☐ False

4. The area element in polar coordinates is

   ☐ $r\,dr\,d\theta$

   ☐ $dr\,d\theta$

   ☐ $\sin\theta\,dr\,d\theta$

   ☐ $r^2\,dr\,d\theta$

5. Evaluate $\iint_D (x^2 + y^2)\,dA$ over the unit disc D, to three decimal places.

   _____

6. If f is differentiable at a point and its gradient there is zero, what follows?

   ☐ The function is constant in a neighbourhood

   ☐ There is one unique steepest direction

   ☐ No directional derivative exists

   ☐ Every unit directional derivative there is zero

7. For $g(x, y) = xy/(x^2 + y^2)$ away from the origin, what is $g(t, t)$ for nonzero t?

   _____

8. For $f = x^2 + 3y^2$, $x = t^2$ and $y = t$, find the path derivative at $t = 1$.

   _____

9. Which reverses the order of $\int_0^1 \int_0^x 1\,dy\,dx$ over the same triangle?

- ☐ $\int_0^1 \int_0^y 1\,dx\,dy$
- ☐ $\int_0^1 \int_1^y 1\,dx\,dy$
- ☐ $\int_0^1 \int_y^1 1\,dx\,dy$
- ☐ $\int_0^1 \int_0^1 1\,dx\,dy$

10. The map $x = 2u$, $y = -3v$ has determinant -6. What positive area factor belongs in a double-integral substitution?

_____

GRE Subject Test · GRE Mathematics

1. $2xy$

   Treat y as fixed: differentiating x^2y with respect to x gives $2xy$. Differentiating y as well would not be this partial derivative.

2. **4.4**

   $\nabla f = (2, 4)$, $\mathbf{u} = (0.6, 0.8)$: $1.2 + 3.2 = 4.4$.

3. **False**

   $x^2 + y^2$ has zero mixed partial.

4. $r\,dr\,d\theta$

   With radius r nonnegative, the determinant and absolute area factor of the standard polar map are r.

5. **1.571**

   $\int_0^{2\pi} \int_0^1 r^3\,dr\,d\theta = \pi/2$.

6. **Every unit directional derivative there is zero**

   The dot product with the zero gradient is zero for every unit direction. Higher-order changes may still occur, so local constancy does not follow.

7. **0.5**

   The numerator is t^2 and the denominator is $2t^2$, giving 1/2. This conflicts with an assigned origin value of zero when checking continuity.

8. **10**

   The composite is $t^4 + 3t^2$; its derivative is $4t^3 + 6t$, giving 10.

9. $\int_0^1 \int_y^1 1\,dx\,dy$

   For a fixed y between 0 and 1, the region has x between y and 1. Both orders describe $0 \le y \le x \le 1$.

10. **6**

    The absolute determinant is 6. The negative sign concerns orientation rather than a negative area.