**Starter Quiz** ·

GRE Subject Test · GRE Mathematics  Name: _____  Score: _____

1. At $x = 0$, the function $f(x) = |x|$ is

   ☐ continuous but not differentiable

   ☐ neither continuous nor differentiable

   ☐ differentiable but not continuous

   ☐ continuous and differentiable

2. Find the derivative of $\ln(x^2 + 1)$ at $x = 1$.

   _____

3. If $F(x) = \int_0^{x^2} e^t \, dt$, find $F'(1)$ to two decimal places.

   _____

4. Evaluate $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \left(\frac{k}{n}\right)^2$.

   ☐ $\frac{1}{3}$

   ☐ $0$

   ☐ The limit diverges.

   ☐ $\frac{1}{2}$

5. Although $1/k \to 0$, the series $\sum_{k=1}^{\infty} 1/k$ diverges.

   ☐ True  ☐ False

6. For $x \neq 1$, $f(x) = (x^2 - 1)/(x - 1)$, and $f(1) = 5$. Find $\lim_{x\to 1} f(x)$.

   _____

7. If $y = \sin(x^2)$, find $y''(0)$.

   _____

8. Why does the mean value theorem not guarantee a point with zero derivative for $f(x) = |x|$ on $[-1, 1]$?

   ☐ The endpoint values are unequal

   ☐ The interval is not closed

   ☐ The function is not continuous

   ☐ The function is not differentiable at the interior point 0

_____

10. Find $\lim_{n\to\infty} \dfrac{2}{n} \sum_{k=1}^{n} (2 + 2k/n)$.

    ☐ 6

    ☐ 4

    ☐ 8

    ☐ 2

1. **continuous but not differentiable**

   The one-sided derivatives are $-1$ and 1.

2. **1**

   $\frac{2x}{x^2+1}$ at $x = 1$ is 1.

3. **5.44**

   $F'(x) = 2xe^{x^2}$, so $F'(1) = 2e$.

4. $\frac{1}{3}$

   It is $\int_0^1 t^2\,dt = 1/3$.

5. **True**

   True. Terms tending to 0 is necessary but not sufficient; the harmonic series diverges (compare with $\int_1^\infty dx/x$ or group terms).

6. **2**

   Away from 1 the expression equals x+1. Its limit is 2, independently of the assigned value 5.

7. **2**

   $y\prime = 2x\cos(x^2)$, so $y\prime\prime = 2\cos(x^2) - 4x^2\sin(x^2)$. Substituting zero gives 2.

8. **The function is not differentiable at the interior point 0**

   The function is continuous and has equal endpoint values, but the required derivative does not exist at 0. The theorem therefore cannot be applied.

9. **2.72**

   $B\prime(x) = 2xe^{x^2} - e^x$, so $B\prime(1) = e \approx 2.72$. Equal bounds at a point do not make the derivative zero.

10. **6**

    Use width $2/n$ and right endpoints $2 + 2k/n$ on $[2, 4]$. The limit is $\int_2^4 t\,dt = 6$.