

8.1 Differential equations and initial conditions

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS initial condition • characteristic root

Objective

Solve separable first-order equations; Solve constant-coefficient second-order equations; Use initial conditions and uniqueness conditions.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

1 Key knowledge and worked examples

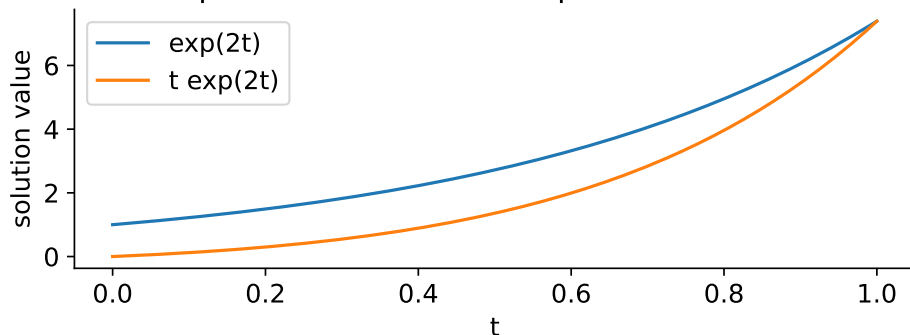
initial condition : A value of the solution or derivative specified at a starting point.

characteristic root : A root of the polynomial governing exponential solutions.

For $y' = ky$, separation and integration give $y = Ce^{kt}$; recover the zero solution if division by y was used. An initial value determines C .

Solve $y'' + 4y = 0$ with $y(0) = 3$ and $y'(0) = 4$. The characteristic roots are $\pm 2i$, so $y = A \cos 2t + B \sin 2t$. The first condition gives $A = 3$; differentiating gives $y'(0) = 2B = 4$, so $B = 2$.

Repeated root 2: two independent solutions



A repeated root 2 gives the two independent solutions e^{2t} and $t \cdot e^{2t}$

2 Practice

2.1 For $y' = 2y$ and $y(0) = 3$, the solution is: [1]

- A $3e^{(2t)}$
- B $2e^{(3t)}$
- C $3 + 2t$

2.2 For a repeated root $r = -1$, the general solution includes: [1]

- A $C_1 e^{(-t)} + C_2 e^{(-t)}$
- B $(C_1 + C_2 t) e^{(-t)}$
- C $C_1 \cos t + C_2 \sin t$

2.3 Solve $y'' - 4y' + 4y = 0$ with $y(0) = 1$, $y'(0) = 0$. Check both initial conditions and the equation. [4]

3 Exam-style questions

3.1 Show that the initial-value problem $y' = 2\sqrt{|y|}$, $y(0) = 0$ has more than one solution for $t \geq 0$. Identify the missing standard uniqueness hypothesis. [4]
