

5.1 Real analysis and topology

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS compact • supremum

Objective

Apply sequence and function limit definitions; Distinguish compactness, connectedness and completeness; Use metric-space and elementary topological reasoning.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

1 Key knowledge and worked examples

compact : Every open cover has a finite subcover.

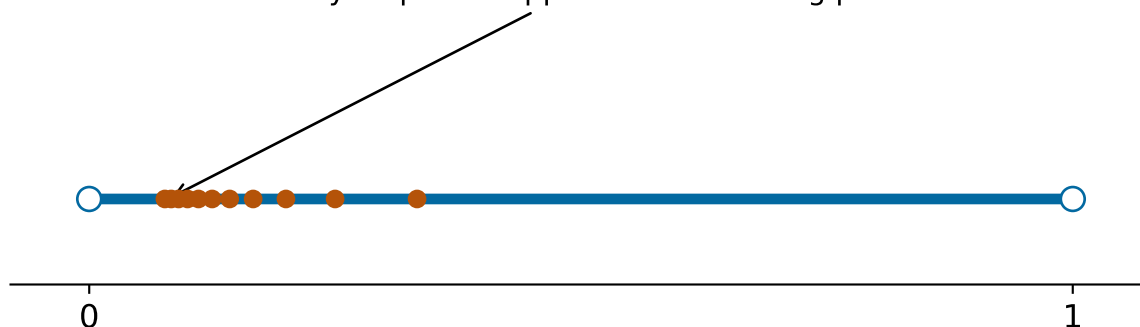
supremum : The least upper bound of a set.

An epsilon–delta statement controls all sufficiently close inputs. The quantifier order matters.

$f(x)=x$ on $(0,1)$ is continuous and bounded but never equals its supremum 1. On $[0,1]$, the same function attains its maximum at 1. The missing endpoint explains why the compact-domain theorem does not apply to the first case.

$X = (0, 1)$: bounded, but incomplete

Cauchy sequence approaches a missing point



$(0, 1)$ is bounded but not complete: a Cauchy sequence can approach a point outside it

2 Practice

2.1 Which subset of the real line is compact? [1]

- **A** $(0,1)$
 - **B** $[0,1]$
 - **C** $[0,\text{infinity})$
-

2.2 A convergent real sequence must be: [1]

- **A** bounded
 - **B** strictly increasing
 - **C** made of integers
-

2.3 Prove continuity of $f(x) = x^2$ at 2 using an explicit epsilon–delta choice. Explain separately why it attains a minimum on $[1,3]$. [4]

3 Exam-style questions

3.1 In the metric space $X = (0, 1)$ with ordinary distance, is X bounded? Is it closed in itself? Is it compact or complete? Explain why “closed and bounded implies compact” cannot be used here. [4]
