3.1 Linear algebra

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS rank • eigenvalue

Objective

Relate rank, nullity and solutions of linear systems; Analyse vector spaces and linear transformations; Compute eigenvalues, determinants and diagonalisation conditions.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

1 Key knowledge and worked examples

rank : Dimension of the image of a linear map.

eigenvalue : A scalar satisfying Av=λv for a nonzero v.

A real vector space is closed under its specified addition and scalar multiplication and satisfies the vector-space axioms. A basis is an independent spanning list: every vector has a unique coordinate representation in that list. To test independence of matrix columns, solve the homogeneous system Ac=0; only the zero coefficient vector means independence. A spanning set can contain redundant vectors, so spanning alone does not make a basis.

A=[[1,1],[0,1]] has characteristic polynomial $(1-\lambda)^2$. Eigenvectors satisfy y=0, so its eigenspace has dimension one. Two independent eigenvectors are needed to diagonalise a 2×2 matrix; A is not diagonalizable.

A repeated eigenvalue can have different eigenspace dimensions

Matrix	Eigenvalue	Dimension	Diagonalizable?
[[1, 1], [0, 1]]	1, twice	1	No
[[1, 0], [0, 1]]	1, twice	2	Yes

A repeated eigenvalue gives a diagonalizable matrix only when its eigenspace has full dimension

2 Practice

2.1 A 3-column matrix has rank 2. Its nullity is: [1]

- **A** 1
- **B** 2
- **C** 3
- **D** 0
- **E** 5

2.2 det A=0 implies: [1]

- **A** A is invertible
- **B** A is singular
- **C** All entries are zero
- **D** Every column is zero
- **E** Every eigenvalue is nonzero

2.3 For $T:\mathbb{R}^3\to\mathbb{R}^2$ with $T(x,y,z)=(x+2y+z,2x+4y+2z)$, find bases of the kernel and image. Solve $T(v)=(3,6)$ and $T(v)=(3,7)$. [4]

3 Exam-style questions

3.1 Let $A=\begin{pmatrix}2&1\\0&2\end{pmatrix}$. Determine its eigenvalues, determinant and diagonalisation over $\mathbb{R}$ and $\mathbb{C}$. Find A^n for integers $n\ge 1$ without repeated multiplication. [4]