24.1 Vector geometry, projections and oriented area

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS orthogonal • cross product

Objective

Use dot products to classify angles and compute projections; Calculate triangle area and orientation using a cross product; Construct or rule out planar dot-product sign configurations.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

1 Key knowledge and worked examples

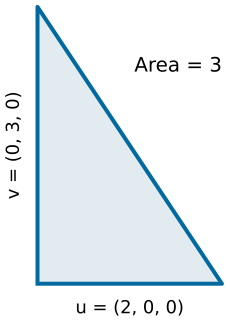
orthogonal : Having a zero dot product in a real inner-product space.

cross product : An oriented perpendicular vector in three dimensions whose magnitude is spanned parallelogram area.

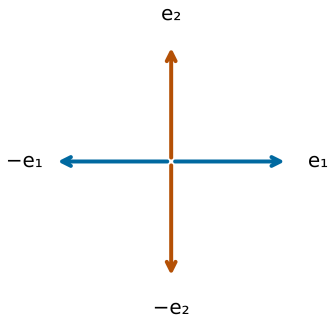
For nonzero real vectors u,v, $u \cdot v = |u||v| \cos\theta$. A positive, zero or negative dot product corresponds to an acute, right or obtuse smaller angle. The scalar projection of v along u is $(v \cdot u)/|u|$; its vector projection is $((v \cdot u)/(u \cdot u))u$. The difference from this projection is orthogonal to u. Do not confuse a projected vector with its signed scalar component. The zero vector is orthogonal to every vector but has no defined angle direction.

With p=(0,0,0), q=(2,0,0), r=(0,3,0), the edges are u=(2,0,0), v=(0,3,0). Their cross product is (0,0,6), giving triangle area 3. For w=(3,4) and u=(1,0), the vector projection is (3,0) and the orthogonal remainder is (0,4). The plane 2x−y+2z=6 has normal length 3, so its distance from the origin is 6/3=2.

Triangle area = |u × v| / 2 = 3



Two negative products; four zeros



Triangle area = |u × v| / 2 = 6 / 2 = 3

2 Practice

2.1 Triangle vertices are (0,0,0),(2,0,0),(0,3,0). What is its area? [1]

- **A** 1
- **B** 2
- **C** 3
- **D** 6
- **E** 12

2.2 What is the vector projection of (3,4) onto the line spanned by (1,0)? [1]

- **A** (0,4)
- **B** (3,0)
- **C** (1,0)
- **D** (3/5,4/5)
- **E** 5

2.3 Project $v = (2, 3, 6)$ onto $u = (1, 0, 2)$. Find the area of the triangle with vertices O, u and v. Check the projection residual is orthogonal to u. [4]

3 Exam-style questions

3.1 Can four nonzero vectors in $\mathbb{R}^2$ have all six pairwise dot products strictly negative? Give a proof. How does allowing zero products change the possibilities? [4]