23.1 Trigonometric phase and parametric curves

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS phase angle • parametric curve

Objective

Distinguish amplitude, angular frequency, phase angle and horizontal shift; Eliminate a parameter while retaining its domain and tracing direction; Use parametric derivatives without assuming a vertical tangent is stationary.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

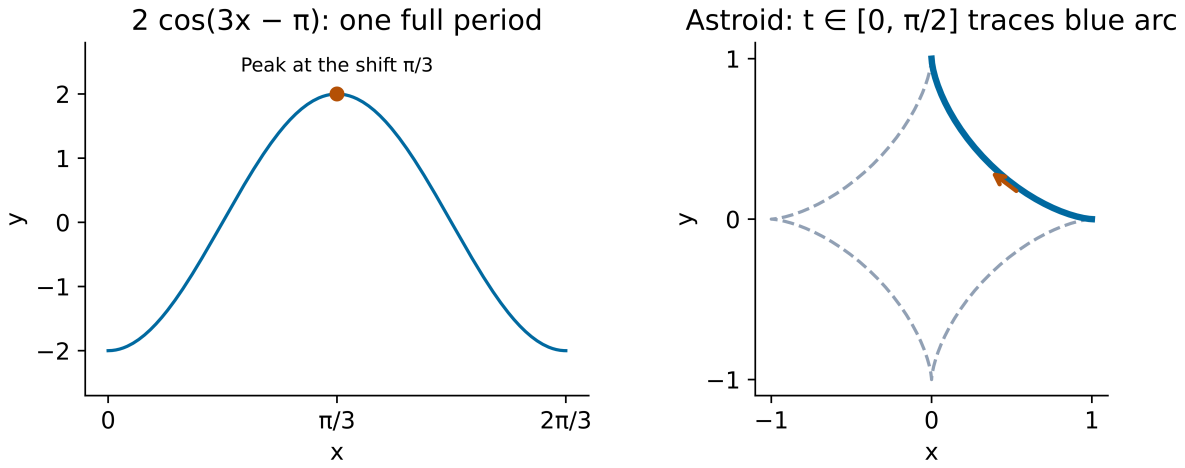
1 Key knowledge and worked examples

phase angle : The angle offset inside a periodic function argument, defined modulo a full period.

parametric curve : A curve whose coordinates are specified as functions of a shared parameter.

For $y=A\cos(\omega x-\varphi)$ with $A>0$ and $\omega>0$, amplitude is A, period is $2\pi/\omega$, and horizontal shift is φ/ω. The phase angle φ is measured inside the cosine argument; it is not itself the horizontal shift unless $\omega=1$. Start with peak-to-peak spacing for the period and midline-to-peak distance for amplitude. To find phase, substitute a known point and check the direction of motion there. Equivalent phases differ by 2π.

For $y=-2\cos(3x)$, minima include $x=0$ and maxima include $x=\pi/3$. The phase representation $2\cos(3x-\pi)$ gives the same values. For $x=2\cos t, y=\sin t$, elimination gives $x^2/4+y^2=1$; at $t=\pi/4$, $dx/dt=-\sqrt{2}$ and $dy/dt=\sqrt{2}/2$, hence slope $-1/2$. The full parameter interval $[0,2\pi]$ traces the ellipse once counterclockwise.



The phase shift $\pi/3$ moves the peak; the period is $2\pi/3$

2 Practice

2.1 For y=2 cos(3x−π), what is the rightward horizontal shift? [1]

- **A** π
- **B** 3π
- **C** $\pi/3$
- **D** $2\pi/3$
- **E** $1/3$

2.2 For x=$\cos^3$t,y=$\sin^3$t with t in [0,$\pi/2$], which set is traced? [1]

- **A** The whole unit circle
- **B** The whole astroid
- **C** Only the first-quadrant astroid arc
- **D** Only the line x+y=1
- **E** A circle of radius 3

2.3 For $y = -3\cos(2x)$, state amplitude, period, a positive-amplitude phase representation and the corresponding horizontal shift. Parameterise the ellipse $x^2/9 + y^2/4 = 1$ counterclockwise. [4]

3 Exam-style questions

3.1 For $x = t^2, y = t^3$ on $-1 \le t \le 1$, determine the traced locus and tangent at t=0. Does vanishing parameter velocity prove there is no tangent? Does the implicit equation alone retain the whole parameter restriction? [4]