21.1 Conditioning, Bayesian inference and sampling error

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS posterior probability • standard error

Objective

Separate conditional, joint and independent-event probabilities; Calculate posterior probabilities with a complete base-rate table; Use variance and sample size to determine a sample mean standard error.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

1 Key knowledge and worked examples

posterior probability : A probability conditional on the observed evidence after accounting for prior proportions.

standard error : The standard deviation of a statistic across repeated samples.

For $P(B)>0$, $P(A|B)=P(A\cap B)/P(B)$: restrict the population to B before computing the fraction in A. Independence means $P(A\cap B)=P(A)P(B)$, equivalently $P(A|B)=P(A)$ when the denominator is nonzero. Mutually exclusive events with positive probabilities cannot be independent, because their joint probability is zero. Sampling without replacement usually changes later probabilities; a fixed denominator for successive draws would describe a different experiment.

Let $P(A)=0.4$, $P(B)=0.5$ and $P(A\cap B)=0.3$. Then $P(A|B)=0.6$, so A and B are not independent, since $0.3\neq 0.4\cdot 0.5$. If a variable has standard deviation 12, an independent sample of size 36 has mean standard error 2. Increasing the sample size to 144 gives 1; the individual-observation standard deviation stays 12.

Condition on positives: 80 / 125, not 80 / 100

Expected group	Positive	Negative	Total
Defective	80	20	100
Not defective	45	855	900
Total	125	875	1000

Given a positive test, divide by all 125 positives: 80/125, not 80/100

2 Practice

2.1 P(A)=0.4, P(B)=0.5 and P(A∩B)=0.3. What is P(A|B)? [1]

- **A** 0.2
- **B** 0.3
- **C** 0.4
- **D** 0.5
- **E** 0.6

2.2 An iid sample of size 25 has mean standard error 6. What is the mean standard error for size 100 from the same population? [1]

- **A** 1.5
- **B** 3
- **C** 6
- **D** 12
- **E** 24

2.3 In a stated model, 10% of devices have a defect. A test is positive for 80% of defective and 5% of nondefective devices. Find the probability of a defect given a positive result using 1000 expected devices. [4]

3 Exam-style questions

3.1 Independent observations have variance 25. Find the standard error of their mean for n=100. If instead every distinct pair has covariance 1, find the mean's variance. Does quadrupling n necessarily halve the standard error in that second model? [4]