

# 20.1 Set images, equivalence relations and logical negation

---

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 10 marks

**KEY WORDS** preimage • equivalence relation

## Objective

---

Prove image identities and distinguish inclusion from equality; Check reflexivity, symmetry and transitivity with explicit cases; Negate implications and quantified statements without changing their scope.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

## 1 Key knowledge and worked examples

---

**preimage** : The set of inputs whose outputs lie in a specified target set.

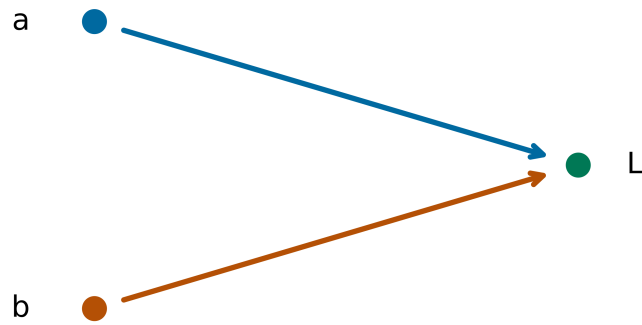
**equivalence relation** : A relation that is reflexive, symmetric and transitive.

For  $f: X \rightarrow Y$  and  $A \subseteq X$ , the image  $f(A)$  consists of all outputs  $f(a)$  with  $a$  in  $A$ . A point is in  $f(A \cup B)$  exactly when it comes from  $A$  or  $B$ , so  $f(A \cup B) = f(A) \cup f(B)$ . If  $A \subseteq B$ , then  $f(A) \subseteq f(B)$ . For an intersection only  $f(A \cap B) \subseteq f(A) \cap f(B)$  is automatic: the same output may come from different inputs. Use  $f(x) = x^2$ ,  $A = \{-1\}$ ,  $B = \{1\}$ ; the left image is empty while the right intersection is  $\{1\}$ .

Let  $f$  map both  $a$  and  $b$  to label  $L$ . With  $A = \{a\}$ ,  $B = \{b\}$ ,  $f(A \cap B) = \emptyset$  but  $f(A) \cap f(B) = \{L\}$ . For  $xRy$  defined by  $x=y$  or  $x=-y$  on  $\mathbb{R}$ , reflexivity and symmetry follow directly, and two sign changes still give  $z = \pm x$ , proving transitivity. The classes are  $\{x, -x\}$ , with  $\{0\}$  a singleton. A false statement “every stored file has a hash” means at least one stored file has no hash; it does not mean all files lack hashes.

Two distinct inputs can share an output

$$A = \{a\}; B = \{b\}$$



$$A \cap B = \emptyset, \text{ but } f(A) \cap f(B) = \{L\}$$

*$f(A \cap B)$  can be empty while  $f(A) \cap f(B)$  is not: images do not preserve intersections*

## 2 Practice

2.1 Which identity holds for every function  $f$  and subsets  $A, B$  of its domain? [1]

- A  $f(A \cap B) = f(A) \cap f(B)$
- B  $f(A \cup B) = f(A) \cup f(B)$
- C  $f(A) = A$
- D  $f(A) \cap f(B) = \emptyset$  whenever  $A \cap B = \emptyset$
- E  $f(A) \subseteq A$

2.2 What is the negation of  $P \Rightarrow (Q \wedge R)$ ? [1]

- A  $\neg P \wedge Q \wedge R$
- B  $\neg P \Rightarrow (\neg Q \wedge \neg R)$
- C  $P \wedge (\neg Q \vee \neg R)$
- D  $P \vee \neg Q \vee \neg R$
- E  $P \wedge \neg Q \wedge \neg R$

**2.3** Prove  $f(A \cap B) \subseteq f(A) \cap f(B)$ . Give a strict-inclusion example. Under what additional assumption on  $f$  does equality follow for all  $A$  and  $B$ ? [4]

---

---

---

---

---

---

### 3 Exam-style questions

**3.1** On the integers define  $aRb$  when  $a - b$  is divisible by 4. Prove it is an equivalence relation and describe every class. Negate: “For every integer  $n$  there exists an integer  $m$  greater than  $n$  with property  $P$ .” [4]

---

---

---

---

---

---

---

---

---

---