

2.1 Multivariable calculus and vector analysis

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS gradient • Jacobian

Objective

Compute partial derivatives, gradients and directional derivatives; Use multiple integrals and coordinate changes; Use derivative conditions to distinguish planes from curved surfaces.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

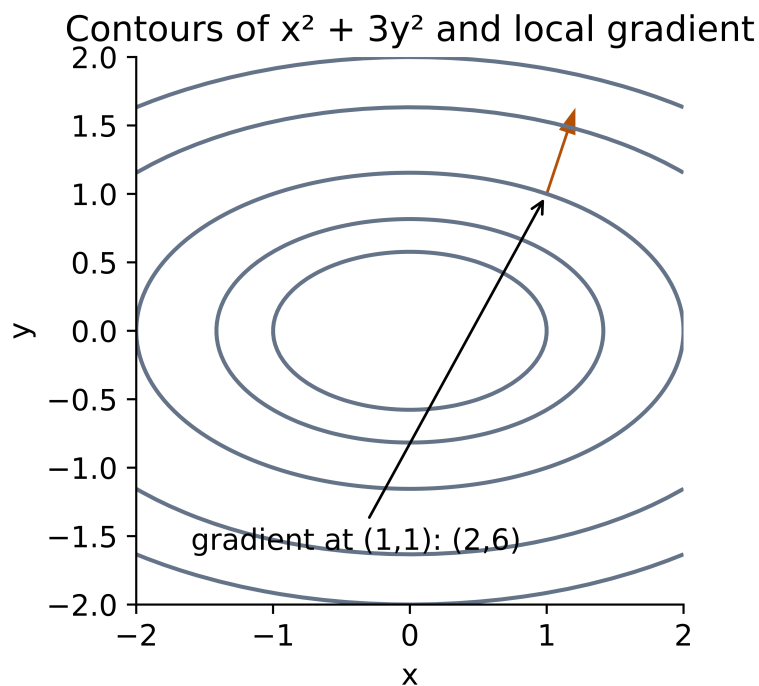
1 Key knowledge and worked examples

gradient : Vector of partial derivatives.

Jacobian : The local area or volume scaling in a coordinate change.

A partial derivative changes one coordinate while fixing the others. For $f = x^2 + 3xy$, $f_x = 2x + 3y$ and $f_y = 3x$. The gradient collects these derivatives; if f is differentiable, the directional derivative along a unit vector $\mathbf{v}$ is $\text{grad } f \cdot \mathbf{v}$. Normalise the direction before taking this dot product. A direction vector of length two would double the answer if used without normalisation.

For $f(x,y) = x^2 + 3y^2$, $\text{gradient } f = (2x, 6y)$. At $(1,1)$ this is $(2,6)$. In direction $(3,4)$, the unit vector is $(3/5, 4/5)$, giving directional derivative $2 \cdot 3/5 + 6 \cdot 4/5 = 6$.



The gradient (2, 6) at (1, 1) is perpendicular to the contour and points uphill

2 Practice

2.1 Partial derivative with respect to x of x^2y is:

[1]

- A x^2
- B $2xy$
- C $2x$

2.2 The polar area element is:

[1]

- A $dr d\theta$
- B $r dr d\theta$
- C $r^2 dr d\theta$

2.3 For $f(x, y) = x^2 + xy + 2y^2$, find the directional derivative at $(1, -1)$ towards $(3, 4)$. Evaluate $\iint_D (x^2 + y^2) dA$ for the unit disk D . [4]

3 Exam-style questions

3.1 Define $g(x, y) = xy/\sqrt{x^2 + y^2}$ away from the origin and $g(0, 0) = 0$. Both partial derivatives at the origin are zero. Is g differentiable there? Is it continuous? [4]
