

# 19.1 Functions, inverse branches and composition

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 10 marks

KEY WORDS bijection • involution

## Objective

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Distinguish injectivity and surjectivity using the stated domain and codomain; Choose and verify an inverse branch by composing in both directions; Trace repeated function composition while preserving the domain.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

## 1 Key knowledge and worked examples

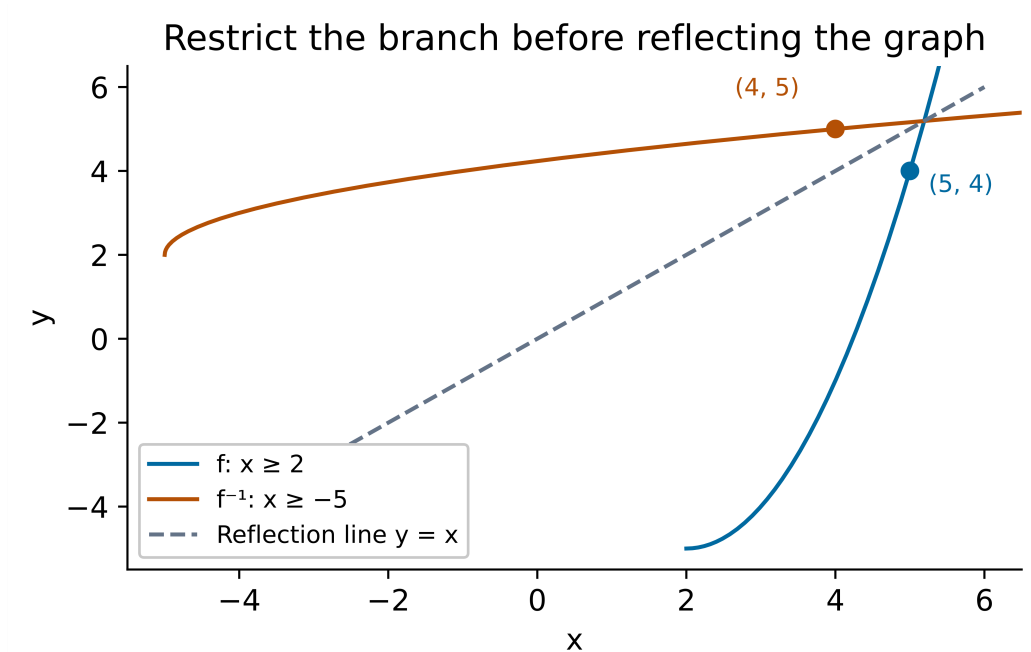
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**bijection** : A function that is both injective and surjective between its specified sets.

**involution** : A function whose composition with itself is the identity on its domain.

A function assigns exactly one output to each allowed input. Injective means equal outputs force equal inputs; surjective means every element of the stated codomain is reached. The map  $x \mapsto x^2$  from  $\mathbb{R}$  to  $[0, \infty)$  is surjective but not injective. From  $[0, \infty)$  to  $[0, \infty)$  it is both, hence bijective. From  $[0, \infty)$  to  $\mathbb{R}$  it remains injective but fails surjectivity. Keep domain, image and codomain separate when testing each claim.

For  $f(x) = (x-2)^2 - 5$  with  $x \geq 2$ ,  $f(5) = 4$  and  $f^{-1}(4) = 2 + \sqrt{9} = 5$ . The unrestricted quadratic would have two inputs,  $-1$  and  $5$ , giving output  $4$ . For  $g(x) = 1/(1-x)$ , the orbit  $3 \rightarrow -1/2 \rightarrow 2/3 \rightarrow 3$  has period three, so  $g^8(3) = 2/3$ ; the count concerns compositions, not eighth powers.



Restrict to one branch before reflecting:  $(5, 4)$  on  $f$  becomes  $(4, 5)$  on  $f^{-1}$

## 2 Practice

2.1 For  $f: [2, \infty) \rightarrow [-5, \infty)$ ,  $f(x) = (x-2)^2 - 5$ , what is  $f^{-1}(y)$ ?

[1]

- A  $2 + \sqrt{y+5}$
- B  $2 - \sqrt{y+5}$
- C  $\sqrt{y-5}$
- D  $1/((y-2)^2 - 5)$
- E  $(y+2)^2 - 5$

2.2 For  $g(x) = 1/(1-x)$  on  $\mathbb{R}$  excluding 0 and 1, what is  $g^8(3)$ ?

[1]

- A 3
- B  $-1/2$
- C  $2/3$
- D  $1/256$
- E 9

**2.3** Restrict  $f(x) = (x-1)^2 + 2$  to  $x \geq 1$ . State its image and inverse, including domains. Verify both inverse compositions. [4]

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### 3 Exam-style questions

**3.1** Let  $h(x) = 1/(1-x)$ . Determine the domain of  $h \circ h \circ h$  and its formula. May the simplified expression be extended to the excluded points without changing the original composite? [4]

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