

1.1 Single-variable calculus and applications

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS continuity • convergence

Objective

Use limits, continuity and differentiability; Apply derivatives, integrals and the fundamental theorem; Recognise Riemann sums and distinguish them from infinite series.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

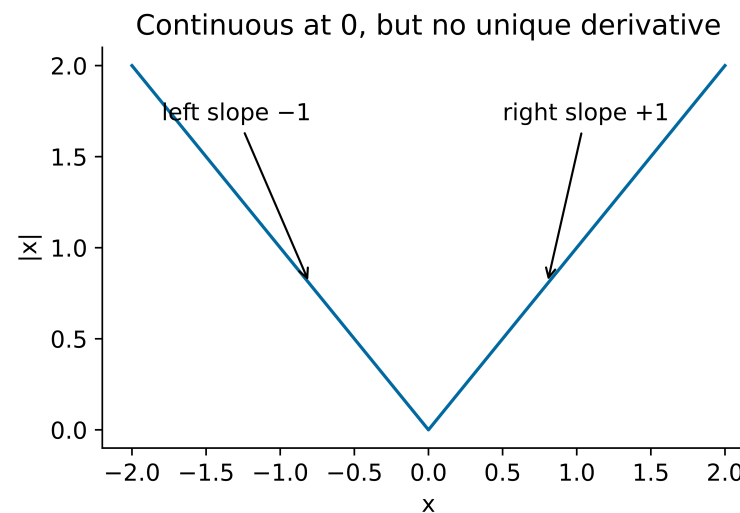
1 Key knowledge and worked examples

continuity : Agreement of a function value with its limit.

convergence : Approach to a finite limiting value.

A limit describes values near a point, without requiring the function to be defined there. Continuity adds the requirement that the function value exists and agrees with the limit. Algebraic cancellation is valid only away from the cancelled zero, but can reveal a removable limit. One-sided limits must agree for a two-sided limit. For quotient limits, check the denominator and hypotheses before applying a rule; $0/0$ is an indeterminate form, not an answer.

For $F(x) = \int_0^x e^t dt$, the fundamental theorem and chain rule give $F'(x) = e^x \cdot 2x$. At $x=1$, $F'(1)=2e$. The upper limit is x^2 , so omitting $2x$ misses its rate of change.



$|x|$ is continuous at 0, but the one-sided slopes -1 and $+1$ disagree, so there is no derivative

2 Practice

2.1 $f(x)=|x|$ at 0 is:

[1]

- **A** continuous and differentiable
- **B** continuous but not differentiable
- **C** neither

2.2 The series $\sum 1/n$ for $n \geq 1$:

[1]

- **A** converges because terms tend to zero
- **B** diverges
- **C** has finitely many terms

2.3 Let $F(x) = \int_x^{x^2} (1+t^2) dt$. Find $F'(x)$ and $F'(2)$. State why the theorem applies. [4]

3 Exam-style questions

3.1 Find $\lim_{n \rightarrow \infty} \sum_{k=1}^n k/(n^2 + k^2)$. Explain why this is a Riemann-sum limit. Then decide whether $\sum_{k=1}^{\infty} 1/k$ converges. [4]
