

6.1 Discrete mathematics, probability and numerical methods

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS recurrence • mutually exclusive

Objective

Use counting, recurrences, graph reasoning and logic; Calculate probabilities and distribution properties; Apply numerical approximation and assess error.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

1 Key knowledge and worked examples

recurrence : A rule linking a sequence term to earlier terms.

mutually exclusive : Events that cannot happen together.

Choose the counting model first: ordered selections of r distinct objects use $n(n-1)\cdots(n-r+1)$, while unordered subsets use $C(n,r)$. A complete simple graph on n vertices has one edge per unordered vertex pair, giving $n(n-1)/2$ edges; loops and multiple edges would change the model. For n lines in general position in a plane, the k th line crosses the prior $k-1$ lines in distinct points and adds k regions. The total is $1+n(n+1)/2$; parallels or triple concurrence invalidate that count.

For $f(x)=x^2-2$ and $x_0=1$, Newton’s step gives $1-(1-2)/2=1.5$. The next value is $1.5-0.25/3=1.4167$. These approximate $\sqrt{2}$, but $f'(0)=0$ makes zero an invalid starting point.

Four labelled independent draws, five equally likely symbols

Event	Count model	Count
ordered strings	$5 \times 5 \times 5 \times 5$	625
all distinct	$5 \times 4 \times 3 \times 2$	120
at least one repeat	$625 - 120$	505

Count “at least one repeat” as the complement of “all distinct”

2 Practice

2.1 How many unordered pairs can be chosen from 5 objects? [1]

- A 10
- B 20
- C 25

2.2 A tree has 8 vertices. Its edge count is: [1]

- A 7
- B 8
- C 9

2.3 Four labelled draws are independent and uniform from five symbols, with replacement. Find the probability that at least two draws agree. Compare the number of ordered and unordered selections of four distinct symbols. [4]

3 Exam-style questions

3.1 Apply bisection to $f(x) = x^3 - 2$ on $[1, 2]$. Give the first two retained brackets and a bound for midpoint error after ten bisections. Would a sign change alone justify bisection for $1/x$ on $[-1, 1]$? [4]

Solutions

2.1 A. 5 choose 2 = $5 \times 4 / 2 = 10$. Twenty counts ordered distinct pairs, counting each twice. Twenty-five permits repeated objects and order.

2.2 A. A finite tree has $n-1$ edges. Eight or nine edges would force a cycle in a connected graph with eight vertices; a finite tree has exactly seven.

2.3 There are 5^4 equally likely ordered draw strings. The no-repeat strings number $5 \cdot 4 \cdot 3 \cdot 2 = 120$.

$$P(\text{repeat}) = 1 - \frac{5 \cdot 4 \cdot 3 \cdot 2}{5^4} = \frac{101}{125}.$$

Ordered distinct selections number 120; unordered subsets number $\binom{5}{4} = 5$. Each subset occurs in $4! = 24$ orders. Dividing by $4!$ is appropriate only for the unordered count, not the draw probability denominator.

3.1 The polynomial is continuous; $f(1) < 0 < f(2)$. The first midpoint is 1.5 with positive value, retaining $[1, 1.5]$. The next is 1.25 with negative value, retaining $[1.25, 1.5]$. After ten halvings, width is 2^{-10} and the midpoint of that retained bracket differs from the bracketed root by at most $2^{-11} = 1/2048$. The error bound uses the bracket, not a small residual alone. For $1/x$, zero is a singularity and there is no root. Continuity on the bracket fails; opposite endpoint signs do not justify the method.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.