23.1 Trigonometric phase and parametric curves

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS phase angle • parametric curve

Objective

Distinguish amplitude, angular frequency, phase angle and horizontal shift; Eliminate a parameter while retaining its domain and tracing direction; Use parametric derivatives without assuming a vertical tangent is stationary.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

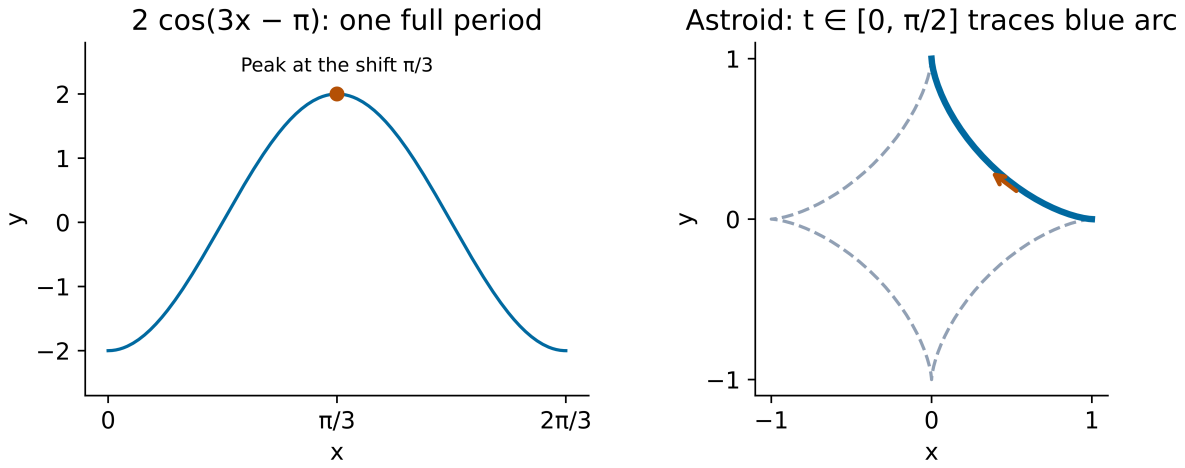
1 Key knowledge and worked examples

phase angle : The angle offset inside a periodic function argument, defined modulo a full period.

parametric curve : A curve whose coordinates are specified as functions of a shared parameter.

For $y=A\cos(\omega x-\varphi)$ with $A>0$ and $\omega>0$, amplitude is A, period is $2\pi/\omega$, and horizontal shift is φ/ω. The phase angle φ is measured inside the cosine argument; it is not itself the horizontal shift unless $\omega=1$. Start with peak-to-peak spacing for the period and midline-to-peak distance for amplitude. To find phase, substitute a known point and check the direction of motion there. Equivalent phases differ by 2π.

For $y=-2\cos(3x)$, minima include $x=0$ and maxima include $x=\pi/3$. The phase representation $2\cos(3x-\pi)$ gives the same values. For $x=2\cos t, y=\sin t$, elimination gives $x^2/4+y^2=1$; at $t=\pi/4$, $dx/dt=-\sqrt{2}$ and $dy/dt=\sqrt{2}/2$, hence slope $-1/2$. The full parameter interval $[0,2\pi]$ traces the ellipse once counterclockwise.



The phase shift $\pi/3$ moves the peak; the period is $2\pi/3$

2 Practice

2.1 For y=2 cos(3x−π), what is the rightward horizontal shift? [1]

- **A** π
- **B** 3π
- **C** $\pi/3$
- **D** $2\pi/3$
- **E** $1/3$

2.2 For x=$\cos^3$t,y=$\sin^3$t with t in [0,$\pi/2$], which set is traced? [1]

- **A** The whole unit circle
- **B** The whole astroid
- **C** Only the first-quadrant astroid arc
- **D** Only the line x+y=1
- **E** A circle of radius 3

2.3 For $y = -3\cos(2x)$, state amplitude, period, a positive-amplitude phase representation and the corresponding horizontal shift. Parameterise the ellipse $x^2/9 + y^2/4 = 1$ counterclockwise. [4]

3 Exam-style questions

3.1 For $x = t^2, y = t^3$ on $-1 \le t \le 1$, determine the traced locus and tangent at t=0. Does vanishing parameter velocity prove there is no tangent? Does the implicit equation alone retain the whole parameter restriction? [4]

Solutions

2.1 C. Factor the argument as 3(x−π/3). The phase π is divided by angular frequency 3. Pi is the phase before division; three pi multiplies rather than divides; two pi over three is the period, and one-third omits the phase pi.

2.2 C. The parameter interval keeps both coordinates nonnegative. Elimination gives the astroid relation, but does not remove this quadrant restriction. The full curves exceed the parameter interval; the circle and line have different equations. The restricted first-quadrant arc alone matches both coordinate signs and the astroid relation.

2.3 Amplitude is three and period $2\pi/2 = \pi$. Write $y = 3\cos(2x - \pi)$, with phase π and rightward shift $\pi/2$. Phase and shift are not the same. One parametrisation is $x = 3\cos t, y = 2\sin t$, $0 \le t \le 2\pi$, which begins at $(3, 0)$ and initially moves upward. Eliminating t checks the ellipse equation.

3.1 Elimination gives $y^2 = x^3$, with $0 \le x \le 1$ and both signs of y. For t nonzero, $dy/dx = 3t/2 \to 0$. The secant slope from the origin is $y/x = t \to 0$ too, so the cusp has horizontal tangent. Both parameter derivatives vanish at zero, making the raw quotient 0/0 inconclusive, not proving absence of a tangent. The unrestricted implicit equation also has points with x>1; the interval restriction must be carried separately.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.