

22.1 Similarity, scaling and conic distance loci

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS similarity • focus

Objective

Match triangle vertices by equal angles before forming a side ratio; Apply length, area and volume scale factors to geometric changes; Identify conic distance conditions and retain a signed hyperbola branch.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

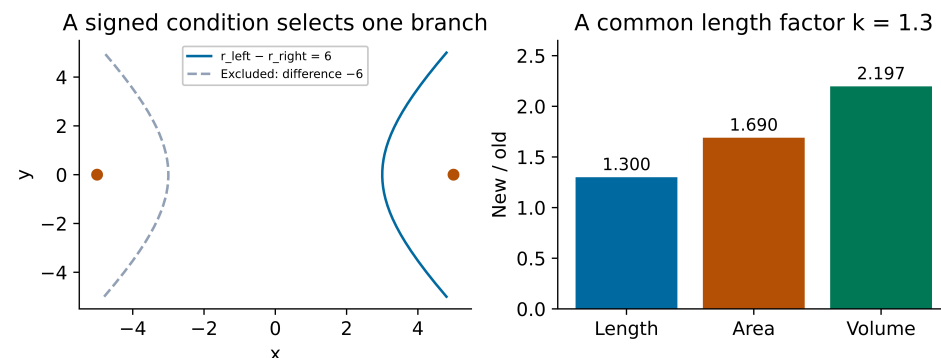
1 Key knowledge and worked examples

similarity : Equality of corresponding angles and a common ratio of corresponding lengths.

focus : A fixed point used in a conic distance definition.

Triangles are similar when their corresponding angles agree; their corresponding side ratios are then equal. Write the vertex correspondence explicitly. If triangle APQ has angle A equal to angle A of ABC and angle P equal to angle C, its ordered correspondence is $A \leftrightarrow A$, $P \leftrightarrow C$, $Q \leftrightarrow B$. Thus $AP/AC = AQ/AB = PQ/CB$. The side AP lies on AB but corresponds to AC; matching by where a side lies or by the letter P can give the wrong ratio.

Let $AB=10$, $AC=15$ and $BC=20$. A point P on AB has $AP=6$, and Q on AC makes angle APQ equal to angle ACB. Then $AP/AC=6/15=2/5$, so $PQ=8$ and $AQ=4$. Separately, foci at $(-5,0)$, $(5,0)$ and distance-to-left minus distance-to-right equal to 6 give $a=3$, $b^2=25-9=16$: $x^2/9 - y^2/16 = 1$ with $x \geq 3$. The point $(-3,0)$ satisfies the squared equation but fails the signed condition.



A signed difference picks one branch; a length factor k scales area by k^2 and volume by k^3

2 Practice

2.1 Correspondence is $A \leftrightarrow A$, $P \leftrightarrow C$, $Q \leftrightarrow B$. If $AP=6$, $AC=15$ and $CB=20$, what is PQ ? [1]

- A 4
- B 6
- C 8
- D 12
- E 50

2.2 A circle radius grows by 30%. What is the area increase? [1]

- A 30%
- B 60%
- C 69%
- D 90%
- E 169%

2.3 Similar solids have surface-area ratio 9:25. Find their corresponding length and volume ratios. Explain why matching vertices matters before forming a triangle ratio. [4]

3 Exam-style questions

3.1 For foci $A = (-5, 0)$ and $B = (5, 0)$, find the locus $|PA| - |PB| = 6$. Derive its standard equation, choose its branch and check the vertex in the unsquared relation. What happens if the requested difference is 12? [4]

Solutions

2.1 C. AP corresponds to AC, not AB. The common scale is $6/15=2/5$; multiply CB=20 by it. The correspondence fixes scale $6/15$. Multiplying twenty gives eight; four, six, twelve and fifty use an unmatched side or wrong operation.

2.2 C. The new area is $1.3^2=1.69$ times the old area, so the increase is 69%, not the full new ratio 169%. Thirty percent uses length change directly; sixty drops the square term; ninety has no area-scale justification; 169 percent is the new percentage of the original, not the increase.

2.3 Taking positive square roots gives length ratio 3:5. Cubing gives volume ratio 27:125. The area ratio is not itself a length scale. Corresponding vertices must match equal angles; otherwise ratios can compare unrelated sides. Similarity is an assumption here; area ratios alone do not prove that arbitrary solids are similar.

3.1 Write $r_A = \sqrt{(x+5)^2 + y^2}$ and $r_B = \sqrt{(x-5)^2 + y^2}$. The original condition is $r_A - r_B = 6$. Since $r_A^2 - r_B^2 = 20x$, multiplying the distance difference by their sum gives $r_A + r_B = 10x/3$. Hence $r_B = 5x/3 - 3$, which must be nonnegative. Squaring this equality gives $(x-5)^2 + y^2 = (5x/3 - 3)^2$. Expanding and collecting gives $16x^2/9 - y^2 = 16$. Thus the half focal distance is $c=5$, half difference is $a=3$, and the hyperbola is $x^2/9 - y^2/16 = 1$, using $b^2 = c^2 - a^2 = 16$. The original positive difference selects the right branch $x \geq 3$. At $(3, 0)$ the distances are 8 and 2, difference six; at $(-3, 0)$ the difference is negative six. Squaring alone retains this invalid branch. A difference of 12 exceeds $|AB| = 10$ and is impossible by the reverse triangle inequality.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.