# 1.1 Single-variable calculus and applications

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

**KEY WORDS** continuity • convergence

## Objective

Use limits, continuity and differentiability; Apply derivatives, integrals and the fundamental theorem; Recognise Riemann sums and distinguish them from infinite series.

Original public practice. Marking guidance is authored locally. Use the topic reference for prerequisites and full theorem statements.

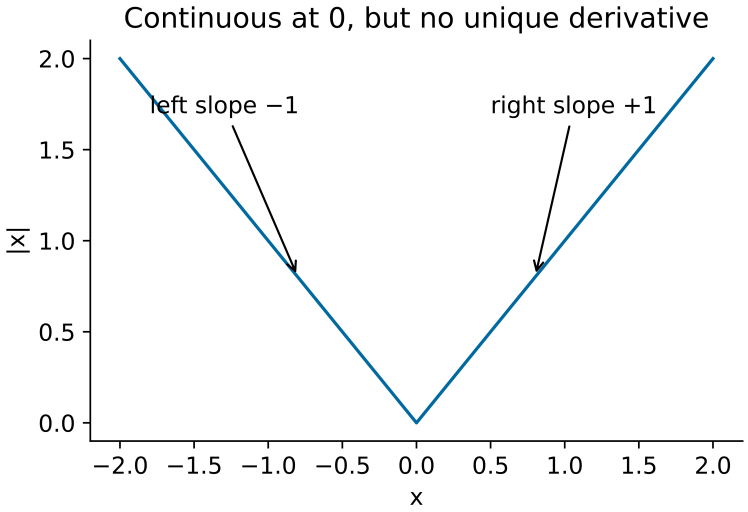
## 1 Key knowledge and worked examples

**continuity** : Agreement of a function value with its limit.

**convergence** : Approach to a finite limiting value.

A limit describes values near a point, without requiring the function to be defined there. Continuity adds the requirement that the function value exists and agrees with the limit. Algebraic cancellation is valid only away from the cancelled zero, but can reveal a removable limit. One-sided limits must agree for a two-sided limit. For quotient limits, check the denominator and hypotheses before applying a rule; 0/0 is an indeterminate form, not an answer.

For F(x)=integral from 0 to $x^2$ of e^t dt, the fundamental theorem and chain rule give $F'(x)=e^{(x^2)} \cdot 2x$. At x=1, $F'(1)=2e$. The upper limit is $x^2$, so omitting 2x misses its rate of change.




*|x| is continuous at 0, but the one-sided slopes −1 and +1 disagree, so there is no derivative*

## 2 Practice

**2.1** f(x)=|x| at 0 is: [1]

- **A** continuous and differentiable
- **B** continuous but not differentiable
- **C** neither

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**2.2** The series sum 1/n for n≥1: [1]

- **A** converges because terms tend to zero
- **B** diverges
- **C** has finitely many terms

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**2.3** Let $F(x) = \int_{x}^{x^2} (1 + t^2)\,dt$. Find $F'(x)$ and $F'(2)$. State why the theorem applies. [4]

## 3 Exam-style questions

**3.1** Find $\lim_{n\to\infty} \sum_{k=1}^{n} k/(n^2 + k^2)$. Explain why this is a Riemann-sum limit. Then decide whether $\sum_{k=1}^{\infty} 1/k$ converges. [4]

## Solutions

**2.1 B**. Left and right derivatives are $-1$ and 1. A fails because one-sided derivatives differ; C fails because the function value and limit are both zero.

**2.2 B**. The harmonic series diverges; the term test alone is insufficient. A confuses a necessary condition with a sufficient one; C fails because there is a term for every positive integer.

**2.3** The integrand is continuous on the real line. The fundamental theorem and chain rule apply at both bounds:

$$F'(x) = (1 + x^4)2x - (1 + x^2).$$

$$F'(2) = (1 + 2^4)\,2 \cdot 2 - (1 + 2^2) = 63.$$

Independently, an antiderivative is $t + t^3/3$. Thus $F = x^2 + x^6/3 - x - x^3/3$, with the same derivative.

**3.1** Rewrite each term as $\frac{1}{n}\frac{k/n}{1+(k/n)^2}$. The continuous function $f(t) = t/(1 + t^2)$ on $[0, 1]$ and mesh $1/n$ give

$$L = \int_0^1 \frac{t}{1 + t^2}\,dt = \frac{1}{2}\ln 2.$$

The harmonic series diverges: the block from $2^{j-1} + 1$ through $2^j$ contains $2^{j-1}$ terms at least $1/2^j$, so each block adds at least $1/2$. Terms tending to zero is necessary but insufficient. The finite-sum integrand here depends on the scaled index; it is not the harmonic series.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.