Exercise walk-through

15.1
Taylor expansions and power-series endpoints

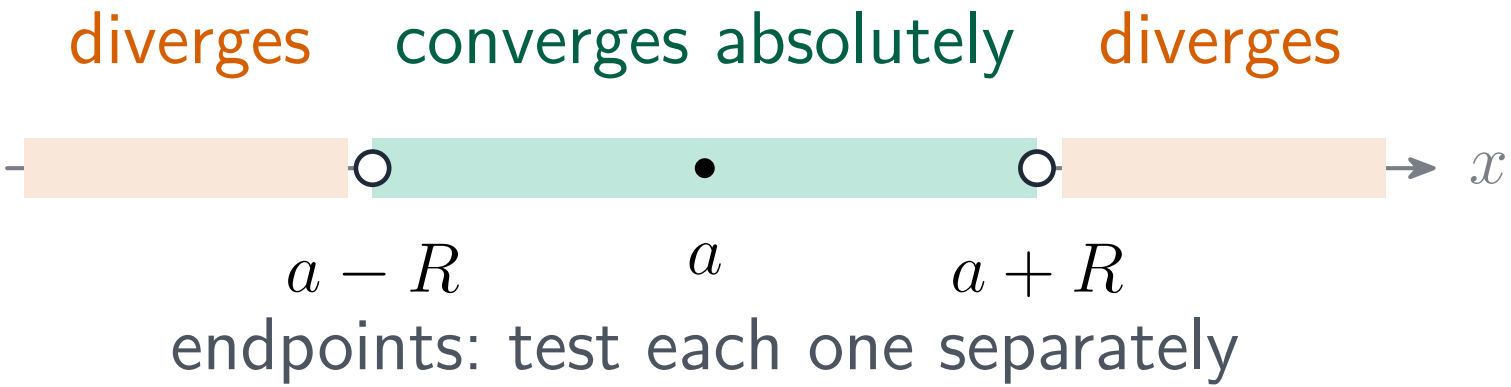
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Taylor expansions and power-series endpoints

- Find Taylor polynomials and bound the **remainder** .
- Find the **radius of convergence** and test the endpoints.
- Differentiate and integrate power series term by term.
- Use standard series to find sums and coefficients.

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Figure: Radius of convergence



- Inside R, outside R, and the endpoints.
- Match each part of the diagram to the text.

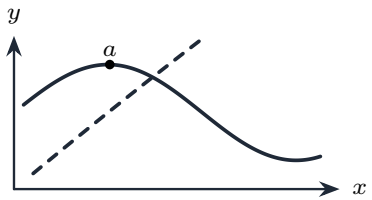
Standard series

- $e^x = \sum x^n/n!$; $\sin x = x - x^3/3! + \cdots$; $\cos x = 1 - x^2/2! + \cdots$, all for every x.
- $\dfrac{1}{1-x} = \sum x^n$ for $|x| < 1$.
- $\ln(1+x) = x - x^2/2 + x^3/3 - \cdots$ for $-1 < x \le 1$.
- $\arctan x = x - x^3/3 + x^5/5 - \cdots$ for $-1 \le x \le 1$.

Exercise 2.1

2.1 What is the coefficient of x^3 in the Maclaurin series of e^(2x)? [1]

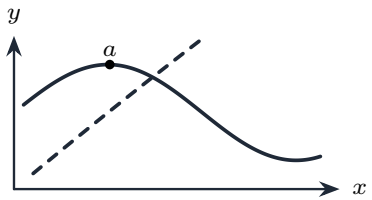
- **A** $1/6$
- **B** $1/3$
- **C** $2/3$
- **D** $4/3$
- **E** 8



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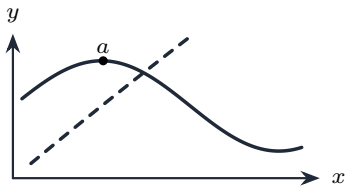


2.1 D. The coefficient is $2^3/3!=8/6=4/3$; the factorial cannot be omitted. One-sixth is the coefficient for e^x; one-third and two-thirds omit some powers of two; eight omits division by factorial three.

Exercise 2.2

2.2 Which is the interval of convergence of sum x^n/n^2 for n≥1? [1]

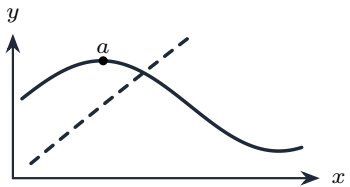
- **A** $(-1,1)$
- **B** $[-1,1)$
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- **D** $[-1,1]$
- **E** R



Exercise 2.2

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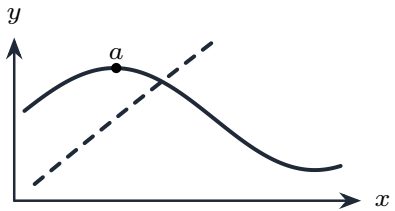
- **A** (−1,1)
- **B** [−1,1)
- **C** (−1,1]
- **D** [−1,1]
- **E** R



2.2 D. The radius is 1 and both endpoints converge absolutely by comparison with sum $1/n^2$. Each open or half-open interval wrongly discards at least one absolutely convergent endpoint; all of R includes inputs with magnitude greater than one where terms fail to tend to zero.

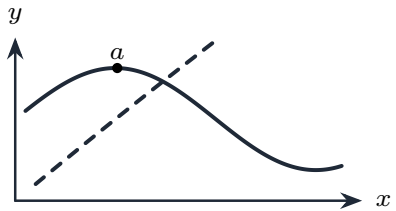
Exercise 2.3

2.3 Find the convergence interval of $\sum_{n=1}^{\infty} x^n/n$ and of its termwise derivative. Approximate $e^{0.1}$ by its quadratic Taylor polynomial with a justified error bound. [4]



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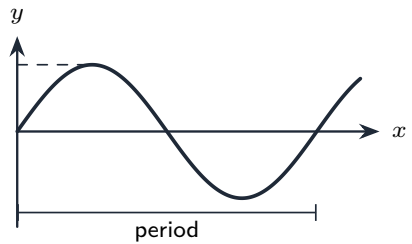


2.3 The ratio test gives radius one. At 1 the original harmonic series diverges; at -1 the alternating series converges. Thus the original interval is $[-1, 1)$. The derivative is $\sum_{n=1}^{\infty} x^{n-1} = 1/(1-x)$ only for $|x| < 1$; its terms do not tend to zero at either endpoint. $P_2(0.1) = 1 + 0.1 + 0.1^2/2 = 1.105$. On $[0, 0.1]$, the third derivative is $e^x < 2$. The Lagrange bound gives $|R_2| \le 2(0.1)^3/6 = 1/3000$. The bound is deliberately conservative, not a claim of exact error.

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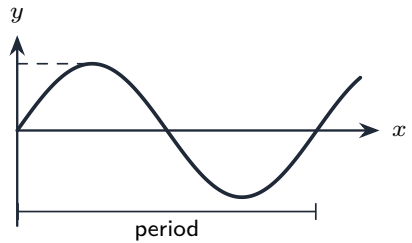
Exercise 3.1

3.1 Evaluate $\lim_{x\to 0}(\cos x - 1 + x^2/2)/x^4$. Explain why replacing cosine by only $1 - x^2/2$ cannot determine this limit. [4]



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3.1 Taylor's theorem near zero gives $\cos x = 1 - x^2/2 + x^4/24 + O(x^6)$. Subtraction cancels the constant and quadratic terms, leaving $x^4/24 + O(x^6)$. After division the limit is $1/24$. A truncation with remainder merely $o(x^2)$ gives no control after division by x^4; retain and bound the first surviving order.

Local four-mark guidance for each written task: choose and justify the method; carry through the reasoning; give every requested result; check the relevant restriction, hypothesis or boundary case. These are classroom marks, not GRE scoring.