Geometry: labelled facts, coordinates, circles and solids

Original classroom teaching; not an official form or score predictor

Geometry: Labelled Facts, Coordinates, Circles and Solids

- Reason from labelled facts, not from how a figure looks.
- Use right triangles, coordinates, the distance formula and the **midpoint** 中点.
- Find arc length and the area of a **sector** 扇形.
- Use a **scale factor** 缩放比例 for lengths, areas and volumes.

Use the given facts

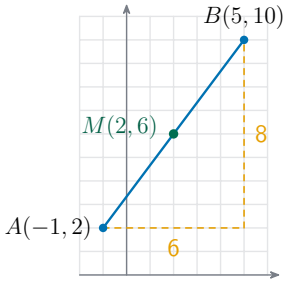
- Unless the question says the figure is to scale, do not measure it or trust apparent right angles, equal lengths or parallel lines.
- Use what is labelled: lengths, angles, "parallel", "perpendicular", "tangent", "centre".
- Points shown on a straight line are on that line, and the order of points shown can be relied on.

Triangles and coordinates

- The distance formula is Pythagoras applied to the horizontal and vertical legs.
- Common right triangles: 3–4–5, 5–12–13, 6–8–10, and $45°$–$45°$–$90°$, $30°$–$60°$–$90°$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M = \left(\tfrac{x_1+x_2}{2}, \tfrac{y_1+y_2}{2}\right)$$

Figure: Triangles and coordinates



$d = \sqrt{6^2 + 8^2} = 10$
M averages the coordinates

- Distance from a right triangle; **midpoint** by averaging.
- Match each part of the diagram to the text.

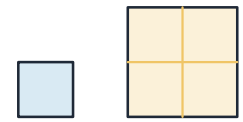
Circles and sectors

- Circumference $2\pi r$; area πr^2.
- A sector with central angle θ° is $\dfrac{\theta}{360}$ of the circle: arc $\dfrac{\theta}{360} \cdot 2\pi r$, area $\dfrac{\theta}{360} \cdot \pi r^2$.
- A tangent is perpendicular to the radius at the point of contact.

Solids and scaling

- Rectangular box $V = lwh$; cylinder $V = \pi r^2 h$, surface area $2\pi r^2 + 2\pi rh$.
- If every length is multiplied by k: areas by k^2, volumes by k^3.
- Work backwards too: an area scale of 9 means a length scale of 3.

Figure: Solids and scaling

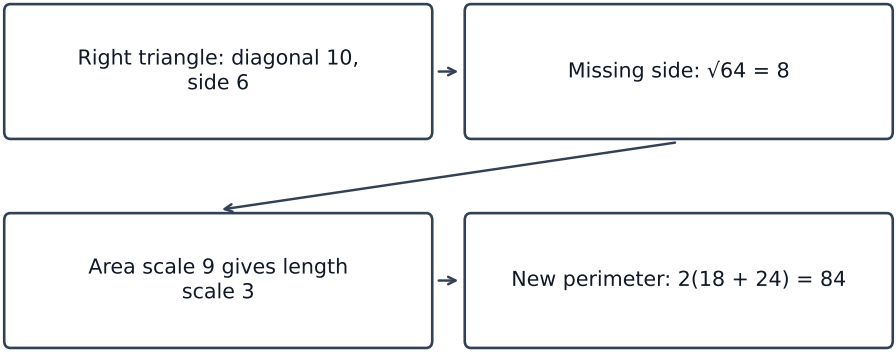


length: $\times k = \times 2$
area: $\times k^2 = \times 4$ (four unit squares)
volume: $\times k^3 = \times 8$

given an area ratio of 4, take the square root: the length ratio is 2

- Length k, area k squared, volume k cubed.
- Match each part of the diagram to the text.

Figure: Solids and scaling ·



- Area scale 9 means length scale 3: the 6 by 8 rectangle becomes 18 by 24

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Worked example: coordinates, a sector and scaling

- $A(-1, 2)$ and $B(5, 10)$: the legs are 6 and 8, so $d = 10$; **midpoint** $(2, 6)$.
- A circle of radius 6 with a 60° sector: area $\frac{60}{360} \cdot 36\pi = 6\pi$; arc $\frac{60}{360} \cdot 12\pi = 2\pi$.
- Enlarging every length by 3 multiplies area by 9 and volume by 27, not by 3.

Worked example: an original application

- Points A=(-1,2) and B=(5,10) are given.
- Find their distance and **midpoint**.
- A right circular cylinder has radius 3 cm and height 4 cm.
- Find its volume and total surface area, including both ends.

Independent transfer

- A rectangle has diagonal 10 cm and one side 6 cm.
- A similar rectangle has area nine times as large.
- Find its perimeter.
- Explain which labelled facts permit Pythagoras, and whether the same calculation would hold for an arbitrary quadrilateral.

Common errors

- Trusting a figure that is not to scale.
- Forgetting the $\frac{\theta}{360}$ fraction for a sector.
- Multiplying area or volume by k instead of k^2 or k^3.

Common errors (2)

- Leaving out one end when finding a cylinder's total surface area.
- Mixing up radius and diameter.

Key terms

- **midpoint**: The point halfway between two points.
- **sector**: The part of a circle between two radii and an arc.
- **scale factor**: The number every length is multiplied by in an enlargement.

Check yourself

- Find the circumference of a circle with radius 5.
- Find the area of a 90° sector of a circle with radius 4.
- A cube's side changes from 2 to 6. By what factor does its volume change?