Algebra: domains, systems, inequalities and functions

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Algebra: Domains, Systems, Inequalities and Functions

- State each **domain restriction 定义域限制** before solving.
- Reject an **extraneous root 增根** created by squaring or by multiplying through.
- Solve inequalities with a **sign chart 符号表**.
- Find the **simultaneous solution 联立解** of a system, and work with functions.

Domains first

- A denominator cannot be zero; an even root needs a non-negative inside.
- Write the restriction before you simplify: $\dfrac{x^2-4}{x-2}$ requires $x \neq 2$, even though it simplifies to $x+2$.
- A solution that breaks a restriction must be rejected.

Equations that create false roots

- Squaring both sides can add a solution: check every candidate in the original.
- For $\sqrt{x+2} = x$: we need $x \geq 0$; squaring gives $x^2 - x - 2 = 0$, candidates 2 and -1; only 2 works.

Inequalities

- Move everything to one side and factor.
- Mark the roots on a number line and test one value in each interval.
- Include the roots for $\leq$ or $\geq$, exclude them for $<$ or $>$.
- Multiplying or dividing by a negative number reverses the sign.

Figure: Inequalities

$(x-1)(x-4)<0$: test one value in each interval



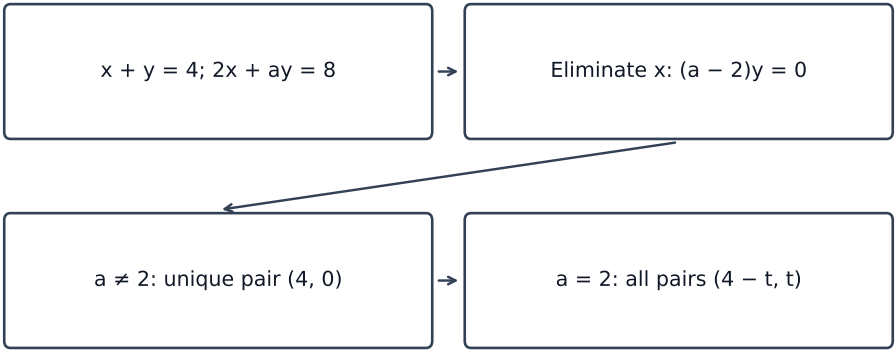
$1<x<4$, open circles: equality not allowed

- Order the roots, test each interval, include endpoints only when equality is allowed.
- Read the axes and units first.

Systems and functions

- Solve linear systems by elimination or substitution, and check in both equations.
- If one equation is a multiple of the other, there are infinitely many solutions; if only the left sides match, none.
- $f(g(x))$: apply g first, then f.

Figure: Systems and functions



- The value of a decides between one solution and infinitely many

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Worked example: four short problems

- $\sqrt{x+2} = x$: only $x = 2$ (check: $\sqrt{4} = 2$; -1 gives $\sqrt{1} = 1 \neq -1$).
- $(x-1)(x-4) \leq 0$: the parabola opens upwards and is at or below zero between its roots, so $1 \leq x \leq 4$.
- $x + y = 10$ and $x - y = 2$: adding gives $2x = 12$, so $x = 6$, $y = 4$.
- $f(t) = 2t + 1$, $g(x) = x^2$: $f(g(3)) = f(9) = 19$, but $g(f(3)) = g(7) = 49$.

Worked example: an original application

- Solve $(x^2-4)/(x-2)=5$ on its original real domain.
- Solve $(x+1)(x-3)\le 0$ and find $f(g(2))$ for $f(t)=t^2$, $g(x)=x-3$.
- The rational equation excludes x=2.
- For allowed x it reduces to x+2=5, giving x=3, which substitutes correctly.

Independent transfer

- For real a, classify the system $x + y = 4$, $2x + ay = 8$ as having one, none or infinitely many solutions.
- Compare with the second equation $2x + ay = 9$.
- Give solutions in every consistent case.
- Subtract twice the first equation.

Common errors

- Simplifying before noting the restrictions.
- Keeping a candidate that fails the original equation.
- Forgetting to reverse an inequality when dividing by a negative.

Common errors (2)

- Including endpoints in a strict inequality.
- Applying the functions in $f(g(x))$ in the wrong order.

Key terms

- **domain restriction**: A value the variable cannot take.
- **extraneous root**: A candidate produced by the method that does not satisfy the original equation.
- **sign chart**: A number line showing where an expression is positive or negative.
- **simultaneous solution**: Values that satisfy all the equations of a system at once.

Check yourself

- Solve $|x-3|<2$.
- For which a do $x+y=4$ and $2x+ay=8$ have infinitely many solutions?
- State the domain of $\dfrac{1}{x-5}$.