Arithmetic: divisibility, remainders and multiplicative change

Original classroom teaching; not an official form or score predictor

Arithmetic: Divisibility, Remainders and Multiplicative Change

- Use **prime factorisation** 质因数分解 to find the **greatest common factor** 最大公因数 and the **least common multiple** 最小公倍数.
- Reason about the **remainder** 余数 when integers are divided.
- Work with ratios and with successive percentage changes.

Factors and multiples

- Write each number as a product of primes: $18 = 2 \cdot 3^2$, $24 = 2^3 \cdot 3$.
- GCF: take each shared prime to its smaller power: $2 \cdot 3 = 6$.
- LCM: take every prime to its larger power: $2^3 \cdot 3^2 = 72$.
- For two numbers, GCF $\times$ LCM $=$ their product: $6 \times 72 = 432 = 18 \times 24$.

Figure: Factors and multiples

$$12 = 2^2 \times 3^1$$

$$18 = 2^1 \times 3^2$$

GCF: smaller powers $2^1 \times 3^1 = 6$

LCM: larger powers $2^2 \times 3^2 = 36$

check: $6 \times 36 = 216 = 12 \times 18$

- Smaller powers for the GCF, larger powers for the LCM.
- Match each part of the diagram to the text.

Remainders

- "n leaves remainder 2 when divided by 5" means $n = 5k + 2$ for some integer k.
- Work with the form: $2n = 10k + 4 = 5(2k) + 4$, remainder 4;
  $3n = 15k + 6 = 5(3k + 1) + 1$, remainder 1.
- A remainder is always at least 0 and less than the divisor.
- Even and odd: even $\pm$ even and odd $\pm$ odd are even; an odd times an odd is odd.

Ratios

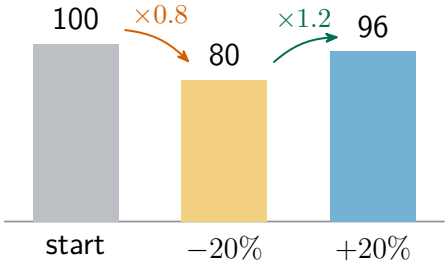
- In a ratio $a : b$, the total has $a + b$ parts.
- Dividing 250 in the ratio $2 : 3$: each of 5 parts is 50, so the shares are 100 and 150.

Multiplicative change

- Successive percentage changes multiply; they do not add.
- A 20% fall followed by a 20% rise gives $0.8 \times 1.2 = 0.96$: a 4% fall overall.
- Each percentage is taken of a different base.

$$P_{\text{final}} = P_{\text{initial}}(1 + r_1)(1 + r_2)$$

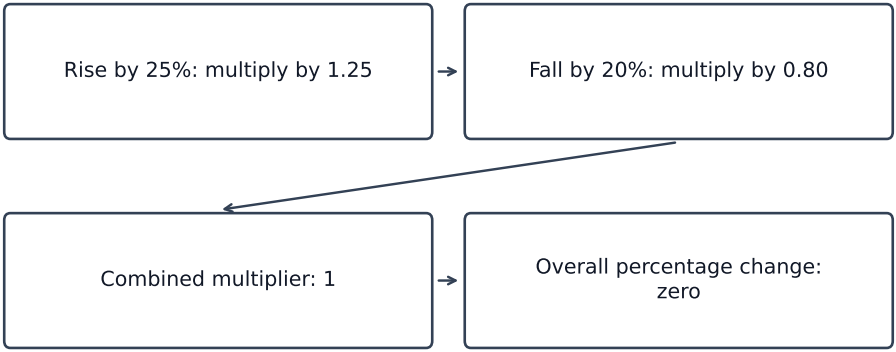
Figure: Multiplicative change



$0.8 \times 1.2 = 0.96$:
not back to 100

- Successive percentages multiply.
- Match each part of the diagram to the text.

Figure: Multiplicative change ·



- $\times 1.25$ then $\times 0.80$ gives $\times 1$, so the overall change is zero

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Worked example: factors and remainders

- $18 = 2 \cdot 3^2$ and $24 = 2^3 \cdot 3$: GCF 6, LCM 72.
- If $n = 5k + 2$, then $2n$ leaves remainder 4 and $n + 4 = 5k + 6 = 5(k + 1) + 1$ leaves remainder 1.
- Test with $n = 7$: $2n = 14 = 2 \times 5 + 4$, and $n + 4 = 11 = 2 \times 5 + 1$.

Worked example: an original application

- Find the **greatest common factor** and **least common multiple** of 36 and 60.
- A price rises 25% then falls 20%.
- Find its overall percentage change.
- $36 = 2^2 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$.

Independent transfer

- An integer n has remainder 5 on division by 8.
- Find the remainder of $3n + 7$ on division by 8, including for negative n.
- A value rises 10% in one year and 20% the next.
- What single percentage reduction restores its original value?

Common errors

- Taking the larger powers for the GCF.
- Giving a remainder equal to or greater than the divisor.
- Adding successive percentages.

Common errors (2)

- Assuming a fall and an equal rise cancel out.
- Treating a part of a ratio as a fraction of the other part.

Key terms

- **prime factorisation**: Writing a number as a product of primes.
- **greatest common factor**: The largest integer that divides each of the numbers.
- **least common multiple**: The smallest positive integer that is a multiple of each of the numbers.
- **remainder**: What is left over after division by a whole number of times.

Check yourself

- Find the GCF of 28 and 42.
- n leaves remainder 3 when divided by 7. What remainder does $n + 5$ leave?
- A price rises 10% and then rises 10% again. What is the overall percentage increase?