Course Companion

GRE General Test

Every topic handout in one volume

全部专题讲义合集

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Verbal: reading and argument structure

GRE General Test

GRE Verbal: Reading and Argument Structure

In 1965 the British medical statistician Austin Bradford Hill gave a lecture in London called "The Environment and Disease: Association or Causation?". Hill and Richard Doll had shown in the 1950s that heavy smokers were far more likely to die of lung cancer, but critics argued that an association was not proof. Hill listed nine points to weigh before calling an association a cause, such as its strength, its consistency across studies, and whether the cause came first. GRE reading passages often make or test the same kind of move: a writer reports a pattern, and you must judge what it does and does not establish.

On the shorter GRE General Test, Verbal Reasoning has 27 questions in two sections. Reading questions ask you to:

- Follow the structure of an argument in connected **discourse** 语篇: claims, evidence and conclusions.
- Identify an **assumption** 假设, the unstated link an argument needs.
- Track whose view each sentence reports.
- Answer single-answer, select-one-or-more and select-in-passage questions exactly as instructed.

Conclusion, evidence and assumption

- Find the conclusion first: what does the writer want you to accept? Words like "therefore" and "so" often mark it.
- The evidence is what is offered in support. The assumption is the gap between them.
- A causal conclusion from timing ("Y rose after X") assumes there is no other change that explains Y.

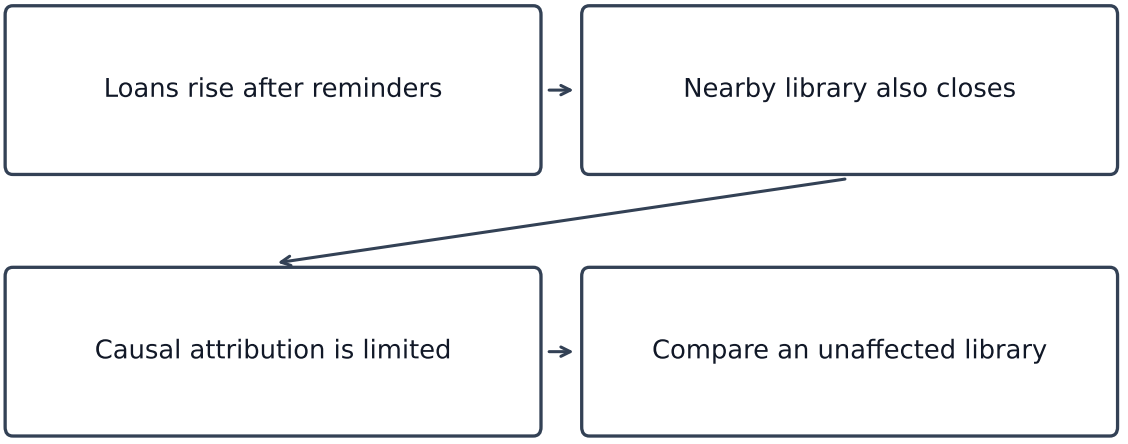
Whose view is this?

- Writers often describe a view before challenging it: "Some historians argue… However…".
- Keep the reported view apart from the author's own.
- The author's attitude shows in words like "promising", "premature", "cannot establish".

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Following the instructions

- Single answer: choose one.
- Select one or more: test every option on its own; there is no partial credit, and one correct option does not make others wrong.
- Select in passage: choose the sentence that does exactly what the question describes, not a nearby one.



Timing alone cannot single out the reminders: a rival change needs a comparison

Worked example: a hidden assumption

"Sales rose after advertising increased. So the advertising caused the rise."

- Evidence: sales rose after advertising increased (timing).
- Conclusion: advertising caused the rise.
- Assumption: no other change, such as a price cut or a competitor closing, explains the rise.
- An option that names a price cut in the same month weakens the argument; one showing sales stayed flat in a similar area without the advertising strengthens it.

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Worked example: an original application

Original passage: [1] A town library's loans rose after it began sending reminders. [2] During the same month, a nearby library closed for repairs. [3] The increase therefore does not establish that reminders alone caused more borrowing. [4] A comparison with an otherwise similar library unaffected by closure would help assess that claim. What is the main purpose? A. Prove reminders never work. B. Limit a causal inference and suggest a comparison. C. Show repairs reduce reading. D. Recommend closing libraries.

Model and reasoning

B. Sentence 3 limits the inference from the observed timing; sentence 4 proposes evidence. A overstates uncertainty as proof of no effect. C changes the outcome from loans to reading and is not established. D turns a reported closure into a recommendation. The author accepts the rise in loans while questioning its explanation.

Independent transfer

For the same passage, select every supported statement: A. Loans increased. B. Reminders caused none of the increase. C. Another change may help explain the increase. Then select the sentence that proposes a way to distinguish explanations.

Check after attempting

A and C; sentence 4. A is stated in sentence 1. C follows from the competing event in sentence 2 and caution in sentence 3. B is unsupported: failure to establish an exclusive cause does not establish zero contribution. Sentence 2 provides a competing observation; sentence 4 proposes a comparison. Judge each option independently and select the complete requested sentence.

Common errors

- Confusing the evidence with the conclusion.
- Giving the author a view the passage only reports.
- Choosing an answer with "always", "never" or "only" when the passage is more cautious.
- In select-all questions, stopping after one correct option.
- Treating "not proved" as "proved false".

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Key terms

discourse: Connected language, such as a passage, with structure and meaning.

assumption: An unstated premise that an argument needs.

Check yourself

- "Crime fell after streetlights were installed, so the lights reduced crime." Name one assumption.
- A passage says a theory "has been challenged but not refuted". Has it been proved?
- In a select-one-or-more question, can two options be correct?

Answers: that nothing else changed at the same time that would reduce crime. No: it has only survived challenges. Yes: test each one separately.

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Verbal: text completion and sentence equivalence

GRE General Test

GRE Verbal: Text Completion and Sentence Equivalence

In 1828 Noah Webster, then aged 70, published *An American Dictionary of the English Language* in two volumes, with about 70,000 entries. He had worked on it for more than twenty years, learning some twenty languages to trace where words came from. Webster knew that many words have several senses and that a sentence chooses one. GRE vocabulary questions test that choice. They do not ask for a definition; they ask which word makes a whole sentence make sense.

Vocabulary questions on the GRE ask you to:

- Complete a sentence or short passage with one, two or three blanks (text completion).
- In sentence **equivalence** 等义关系, choose the two words that each complete a sentence and give it the same meaning.
- Use logic and context to choose a word, such as **arduous** 艰难的 for a task that is long and hard.

Predict before you look

- Cover the options. Read the whole sentence and say, in your own words, what the blank must mean.
- Find the clue: a reason (“because”), a contrast (“although”, “but”), a restatement after a colon, or an example.
- Then choose the option closest to your prediction.

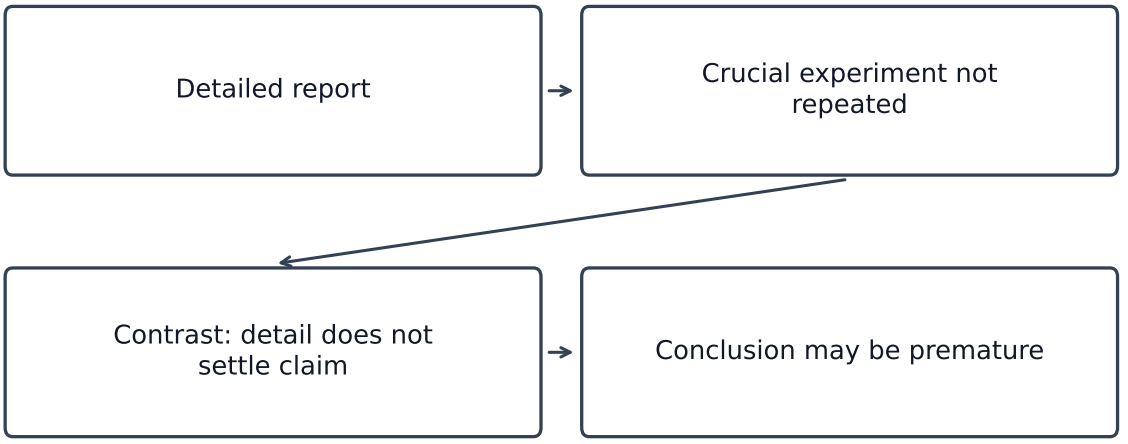
Text completion with several blanks

- Each blank has its own column of three choices. All blanks must be right for the mark.
- Start with the blank that has the clearest clue; then check the others against it.
- Read the finished sentence: the choices must make sense together.

Sentence equivalence

- Choose exactly two of six options. Both must fit the sentence, and both completed sentences must mean the same.
- Two synonyms can both be wrong if neither fits the logic. A word that fits may have no partner with the same meaning.

- Check grammar, tone and logic, not only dictionary meaning.



A detailed report does not settle a claim whose crucial experiment was never repeated

Worked example: a contrast clue

"Although the design looked simple, building it was _____."

- "Although" signals a contrast between how it looked and how it was to build.
- Prediction: "hard work".
- For equivalence, "arduous" and "demanding" both fit and give the same meaning; "easy" and "effortless" are a pair of synonyms that reverse the contrast.

Worked example: an original application

Complete the original sentence: "Although the reviewer admired the report's detail, she found its conclusion _____ because its crucial experiment had not been repeated." A. conclusive B. premature C. exhaustive D. irrelevant E. inevitable

■ **Model and reasoning**

B, premature. The unreplicated crucial experiment makes a settled conclusion too early. Although contrasts admiration for detail with criticism of the conclusion. Conclusive and inevitable oppose the criticism; exhaustive describes completeness, not unsupported timing; irrelevant would require evidence that the conclusion concerns the wrong issue. Predict "too early" before consulting the choices.

Independent transfer

Sentence Equivalence: select exactly two of six choices. "Far from endorsing the proposal without reservation, the committee offered only _____ support." A. qualified B. unconditional C. cautious D. unanimous E. enthusiastic F. immediate

■ Check after attempting

A and C. Both produce support limited by reservations. Unconditional contradicts the explicit contrast. Unanimous concerns how many members agree; immediate concerns timing. Enthusiastic concerns intensity and does not preserve the required limited-support meaning. Contextual fit and equivalent whole-sentence meaning are both necessary; two pleasant-sounding words are insufficient.

Common errors

- Choosing a word because it is difficult or sounds impressive.
- Ignoring a contrast word.
- In sentence equivalence, choosing two synonyms that do not fit.
- Choosing two words that fit but give different meanings.
- In multi-blank items, checking each blank alone, not the whole sentence.

Key terms

equivalence: Here, the requirement that two completed sentences have the same meaning.

arduous: Needing a lot of effort; difficult and tiring.

Check yourself

- "Her praise was _____: she admired the method but doubted the results." Predict a meaning.
- In sentence equivalence, how many options must you choose?
- Can a pair of synonyms be the wrong answer? Why?

Answers: limited or qualified. Exactly two. Yes: if neither word fits the sentence's logic.

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Quantitative comparison and number reasoning

GRE General Test

GRE Quantitative Comparison and Number Reasoning

In 1202 Leonardo of Pisa, later known as Fibonacci, published *Liber Abaci* ("The Book of Calculation"). He had learnt the Hindu–Arabic numerals as a boy in North Africa, where his father was a trade official, and his book taught Italian merchants to calculate with them, including zero. In one problem he accepted a negative answer by reading it as a debt. Thinking about every kind of number, including negative ones, fractions and zero, is exactly what GRE Quantitative Comparison demands.

On the shorter GRE General Test, Quantitative Reasoning has 27 questions in two sections, and an on-screen calculator is available. Quantitative Comparison questions ask you to:

- Compare Quantity A and Quantity B, using every **constraint** 约束条件 given.
- Choose from four fixed answers: A greater, B greater, equal, or cannot be determined.
- Recognise when the relationship is **indeterminate** 不能确定的.
- Use arithmetic, powers, ratios and divisibility to simplify.

The four answers

- (A) Quantity A is always greater; (B) Quantity B is always greater; (C) they are always equal; (D) the relationship cannot be determined.
- "Always" means for every value the conditions allow.
- If you find one case where A is greater and another where B is greater, the answer is (D).

Test the allowed values

- Unless the question says otherwise, a variable can be positive, negative, zero, a fraction or not an integer.
- Try a few kinds of value: 2, -2, 0, $\frac{1}{2}$, a large number.
- Examples can prove (D), but a few examples never prove (A), (B) or (C); for those, you need a reason that covers all cases.

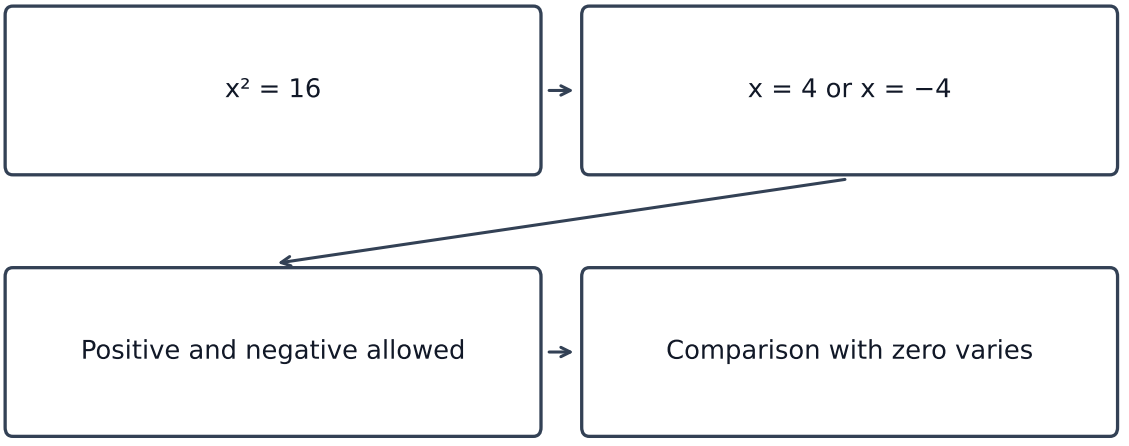
Simplify safely

- You may add or subtract the same amount from both quantities.
- You may multiply or divide both by the same positive number.
- Never divide by a quantity that could be zero or negative without considering those cases.
- Figures are not necessarily drawn to scale: use the given facts.



both allowed, so the relationship cannot be determined

Two allowed values on opposite sides of zero.



$x^2 = 16$ allows $x = 4$ or $x = -4$, so the comparison with zero is not fixed

Worked example: powers of a fraction

$0 < x < 1$. Quantity A: x^2. Quantity B: x^3.

- Try $x = \frac{1}{2}$: $\frac{1}{4} > \frac{1}{8}$, so A is greater.
- Reason for every allowed x: $x^2 - x^3 = x^2(1 - x)$. Both factors are positive when $0 < x < 1$, so A is always greater: answer (A).
- Without the condition, $x = 2$ gives $4 < 8$, so the answer would be (D).

Worked example: an original application

Quantitative Comparison uses these relationships: A. Quantity A is greater. B. Quantity B is greater. C. Equal. D. Cannot be determined. Given real x with $x^2 = 16$, compare Quantity A: x, Quantity B: 0. Repeat with the added condition $x > 0$.

Model and reasoning

Without the added condition, **D**: x=4 makes A greater, while x=-4 makes B greater. With $x > 0$, **A**, because only x=4 is permitted. The principal square root of 16 is four, but the equation has both roots. One chosen example cannot prove a relationship over all allowed values.

Independent transfer

With the same four answer categories, let $a > 0$ and $b < 0$. Compare Quantity A: a/b, Quantity B: b/a. Give admissible examples if the relationship can vary, and identify when equality occurs.

Check after attempting

D. For a=2,b=-1, A=-2 and B=-1/2, so B is greater. For $a = 1$,b=-2, A=-1/2 and B=-2, so A is greater. For $a = 1$,b=-1 they are equal. More generally $a/b = b/a$ means $a^2 = b^2$, and the stated opposite signs give a=-b. Multiplying an inequality by the negative product ab reverses direction; ignoring this sign would reverse conclusions.

Common errors

- Assuming a variable is a positive integer.
- Concluding (A), (B) or (C) from one or two examples.
- Dividing both sides by an expression that could be negative.
- Trusting a figure's appearance.
- Writing $\sqrt{16} = \pm 4$ (the root is 4; the equation $x^2 = 16$ has two solutions).

Key terms

constraint: A condition that limits the allowed values.

indeterminate: Not fixed by the information given.

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Check yourself

- $x \neq 0$. Quantity A: x. Quantity B: $\dfrac{1}{x}$. What is the answer?
- n is a positive integer. Quantity A: $(-1)^{2n}$. Quantity B: 1. What is the answer?
- Why can examples prove (D) but not (A)?

Answers: (D): $x = 2$ gives A greater; $x = \frac{1}{2}$ gives B greater. (C): an even power of -1 is 1. One counterexample shows the relationship varies; "always" needs a reason for every value.

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Quantitative problem solving and data

GRE General Test

GRE Quantitative Problem Solving and Data

In 1763, two years after the death of the English minister Thomas Bayes, his friend Richard Price sent the Royal Society an essay he had found among Bayes's papers. It showed how to reason backwards: given what we observe, how likely is each possible cause? This idea, conditional probability and the theorem named after Bayes, now runs through medicine, law and machine learning. On the GRE it appears in its simplest form: read a table, and use the right denominator.

GRE problem-solving and data questions ask you to:

- Solve problems in arithmetic, algebra, geometry and data analysis; no calculus is needed.
- Use **conditional probability** 条件概率 and percentages of a subgroup.
- Answer in the format asked: one choice, all correct choices, or **numerical entry** 数值填答.
- Check units and the form of the answer before submitting.

Read the question format

- Multiple choice, one answer: choose one.
- Multiple choice, one or more answers: test every option; choose all that work.
- Numerical entry: type a number or a fraction, in the form the box asks for (a decimal, a fraction, or rounded as told).

Conditional probability and subgroups

- "Given that", "of those who", "among" restrict the group. The denominator becomes that group's total.
- $P(A \mid B) = \dfrac{\text{number in both}}{\text{number in } B}$; usually $P(A \mid B) \neq P(B \mid A)$.
- In a percentage question, ask "per cent of what?".

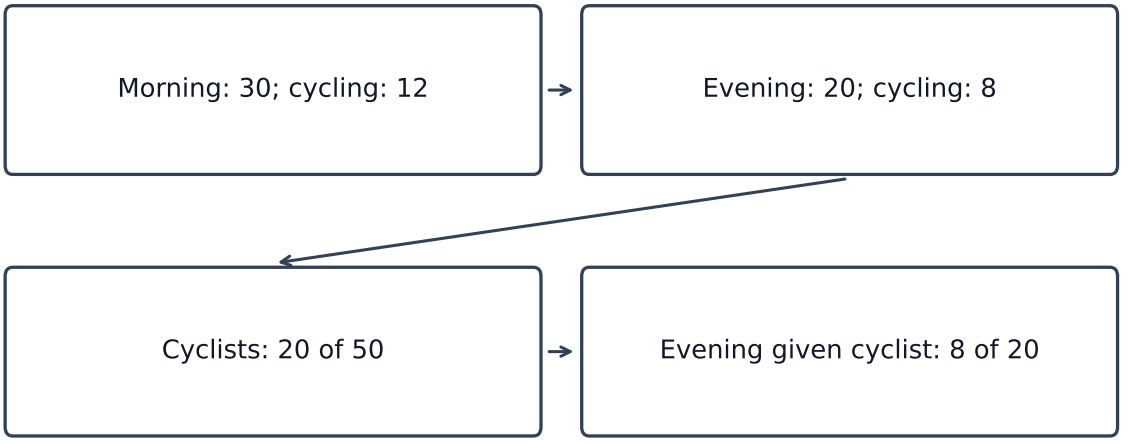


Conditioning shrinks the denominator to the cyclists.

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Estimating and the calculator

- Estimate first, so you can spot a calculator slip.
- Keep exact fractions until the last step, then convert if asked.
- Do not assume a geometric figure is drawn to scale.



Condition on cyclists: 8 of the 20 cyclists travelled in the evening

Worked example: a small table

A class has 12 morning and 8 afternoon students; 3 morning and 4 afternoon students cycle.

- $P(\text{cycles}) = \dfrac{3+4}{20} = \dfrac{7}{20} = 0.35$.
- $P(\text{afternoon} \mid \text{cycles}) = \dfrac{4}{7}$: only the 7 cyclists count.
- $P(\text{cycles} \mid \text{afternoon}) = \dfrac{4}{8} = \dfrac{1}{2}$: a different question with a different denominator.

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Worked example: an original application

In an original survey table, morning students number 30 and evening students 20. Twelve morning and eight evening students cycle. Enter the probability that a randomly chosen student cycles as a decimal. Enter the probability that a cyclist is an evening student as a fraction.

Model and reasoning

Total students are 50 and cyclists 20. The unconditional probability is $20/50 = 0.4$. Restricting to cyclists gives evening probability $8/20 = 2/5$. The denominator changes from 50 to 20 because the second question supplies a condition. An evening student cycling has the reversed probability $8/20$ here only by numerical coincidence: the evening group and cyclist group both happen to have size 20.

Independent transfer

A rectangular display has area 48 cm^2, positive integer side lengths and perimeter at most 32 cm. Select all possible lengths of its longer side: 6, 8, 12, 16, 24. Show why your list is complete.

Check after attempting

8 and 12. Let the longer side be L and shorter side $48/L$. Positive integer factor pairs are (1,48),(2,24),(3,16),(4,12),(6,8). Their perimeters are 98,52,38,32,28 cm. Only the last two meet the bound, with longer lengths 12 and 8. Six is the shorter side of the valid 6-by-8 rectangle, so it fails the requested role. Test both the geometric restriction and the word “longer”; numerical compatibility alone is insufficient.

Common errors

- Using the grand total when the question restricts the group.
- Swapping $P(A \mid B)$ and $P(B \mid A)$.
- Entering a percentage when the box asks for a probability.
- Stopping after one answer in a select-all question.
- Reading a figure as if it were to scale.

Key terms

conditional probability: The probability of an event within a restricted group.

numerical entry: A question format in which you type the answer instead of choosing it.

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Check yourself

- Of 40 students, 10 study French; 6 of those also study Spanish. Find $P(\text{Spanish} \mid \text{French})$.
- The box asks for a probability as a decimal. Do you enter 35 or 0.35?
- A price rises from 80 to 100. What is the percentage increase?

Answers: $\frac{6}{10} = 0.6$. 0.35. $\frac{20}{80} = 25\%$.

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Analytical Writing: Analyze an Issue

GRE General Test

GRE Analytical Writing: Analyze an Issue

In 1859 the English philosopher John Stuart Mill published *On Liberty*. Arguing for free discussion, he wrote: "He who knows only his own side of the case, knows little of that." A person who has never heard the strongest objection to their view, Mill said, cannot really defend it. On the shorter GRE General Test, Analytical Writing is one 30-minute "Analyze an Issue" task, and it rewards exactly what Mill described: a clear position, real reasons, and serious attention to objections.

The Issue task asks you to:

- Take a position on a claim, often a **qualification** 限定条件 of it rather than full agreement or disagreement.
- Follow the specific instruction printed under the claim.
- Support your view with reasons and explained examples.
- Answer a serious **objection** 反对意见 to your view.

Read the instruction

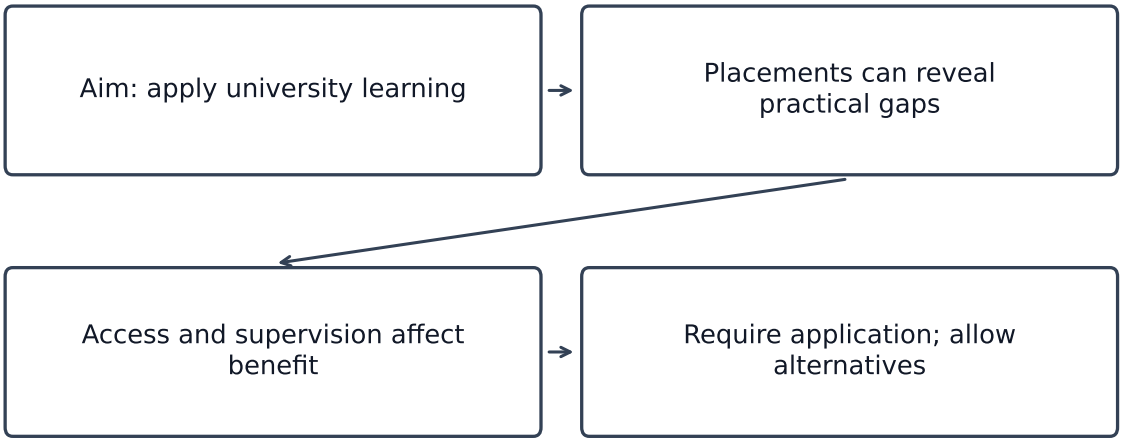
- Every Issue prompt has a claim and an instruction, such as "discuss the extent to which you agree", or "address the most compelling reasons against your position".
- The instruction decides what you must discuss. Underline it.
- Only the Issue task is part of the current standard test; older books may also include a retired "Argument" task.

Choose a position with a scope

- You can agree, disagree or qualify: "This is true for X, but not for Y, because…".
- A qualified position is often stronger because it shows when and why the claim holds.
- State it in one or two sentences in the first paragraph.

Develop reasons and examples

- Each body paragraph makes one point and explains how the example supports it.
- An example alone is not enough; explain the mechanism: why does this policy produce that effect?
- Hypothetical examples are acceptable if they are clearly presented as such.



Weigh the aim against access and supervision before recommending placements

Objections and conclusion

- Choose the strongest objection, not a weak one that is easy to dismiss.
- Answer it, or let it change your position, and say how.
- The conclusion restates the position and its limits. Clear organisation matters more than rare vocabulary.

Worked example: planning in five minutes

Claim: "Universities should require every degree student to complete a work placement."

- Position: support practical application for all students, but not one compulsory form of it.
- Reason 1: in professional subjects, placements reveal the gap between knowing a rule and using it.
- Reason 2: a placement without supervision or relevant work teaches little.
- Objection and answer: unequal access (cost, caring duties) → funded local places and an equally demanding alternative project.

Worked example: an original application

Original Issue prompt: "Universities should require every degree student to complete a work placement." Discuss how far you agree. Explain circumstances in which the recommendation would or would not be useful, using reasons and examples. Plan a qualified position before reading the model.

■ Model and reasoning

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Independent transfer

Without reusing the model's examples, write a paragraph arguing the strongest case for a universal placement requirement. Then explain one condition that weakens that case and revise your thesis accordingly. This is original classroom practice, not an official scored response.

■ Check after attempting

A defensible case is that a common minimum prevents departments from offering application only to students already confident enough to seek it. Develop how universal participation could broaden access and create a clear responsibility for the university. A weakening condition is lack of suitable supervised places: universality alone cannot create educational work. A revised thesis can require application for all while accepting equivalent projects where placements fail the stated educational standard. Other well-developed positions are valid. Review task fit, mechanisms, examples, counterargument and clarity; there is no uniquely correct stance or official score here.

Common errors

- Ignoring the specific instruction.
- Agreeing or disagreeing without saying why.
- Listing examples without explaining how they support the point.
- Raising only a weak objection.
- Spending the time on rare vocabulary instead of clear reasoning.

Key terms

qualification: A limit added to a claim to make it more accurate.

objection: A reason against a position.

Check yourself

- What two parts does every Issue prompt have?
- Is "I partly agree: this works for X but not for Y" an acceptable position?
- What should you do after stating an objection?

Answers: a claim and an instruction. Yes, if you explain the reasons for the limit. Answer it, or show how it changes your position.

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Text completion: dependent meanings across two and three blanks

GRE General Test

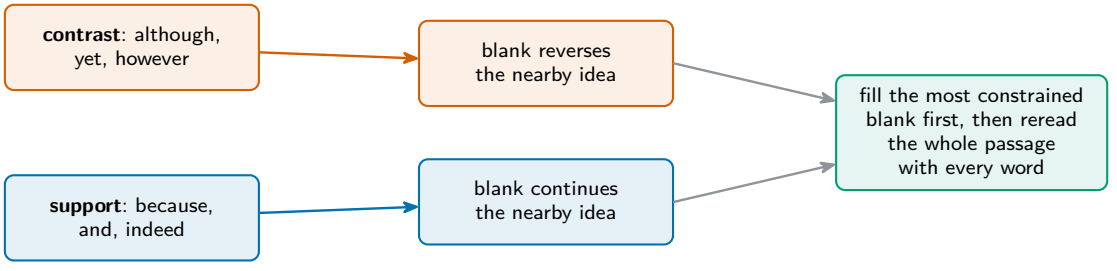
In 1952 Michael Ventris, a 30-year-old English architect, deciphered Linear B, a script found on clay tablets at Knossos in Crete and at Pylos in Greece. Scholars had studied it for fifty years. Ventris built grids of signs that seemed to share sounds, so that each guess constrained the others. When he tried place names on the tablets, the grid suddenly read as an early form of Greek. Each solved sign made the next one easier, and one wrong sign could spoil the whole grid. Multi-blank text completion works the same way: the blanks depend on each other.

These questions ask you to:

- Complete passages with two or three blanks, one choice from each column of three.
- Use cue words and the logic of the whole passage to fix each blank.
- Tell a tentative **conjecture** 推测 from an established claim, and a **dogmatic** 武断的 view from a careful one.
- Check that all the chosen words fit together.

Read the whole passage first

- Read to the end before choosing anything: the clearest clue may come after the blank.
- Mark contrast cues (although, yet, however, despite), support cues (because, so, indeed, and) and limits (only, rather than, not every).
- Predict a plain meaning for each blank in your own words.



Cue words set the direction of each blank.

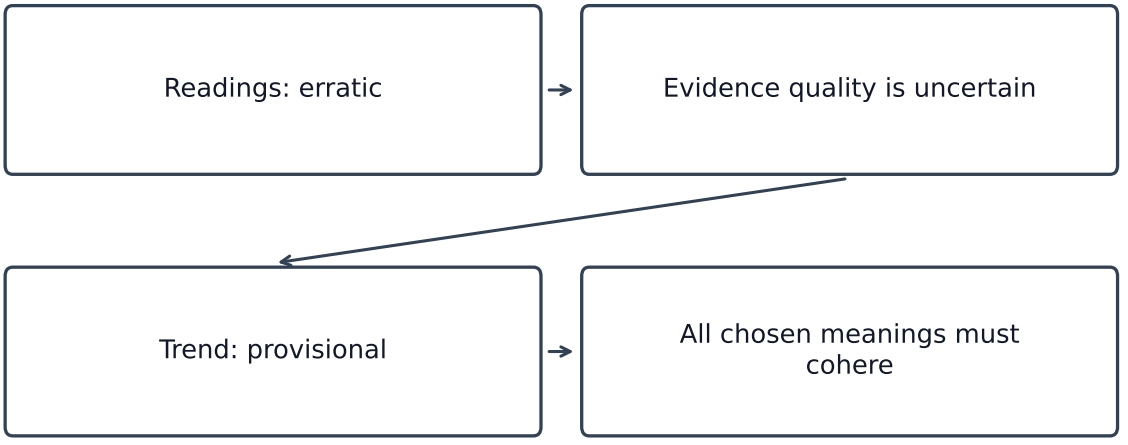
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Start where the clue is strongest

- Fill the most constrained blank first: the one with the clearest cue.
- Use its meaning to narrow the next blank, then reread the first to check it still fits.
- Every blank must be right; there is no credit for two out of three.

Test the finished passage

- Read the passage with all your choices in place.
- Check direction (positive or negative), degree (mild or extreme) and logic (cause, contrast, limit).
- Reject a word that pairs nicely with another blank but contradicts an explicit limit in the passage.



Choose every blank so that the chosen meanings fit together

Worked example: two dependent blanks

"Although the proposal was initially (i) _____, the pilot's consistent results made its critics (ii) _____." Blank (i): plausible / dubious / settled. Blank (ii): reconsider / celebrate failure / lose interest.

- "Although" sets up a contrast between the start and what the results did.
- Consistent results are good news, so the start must be negative: (i) dubious.
- Good results make doubters change their minds: (ii) reconsider.
- Check: "Although the proposal was initially dubious, the pilot's consistent results made its critics reconsider." Direction and logic agree.

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Worked example: an original application

Original two-blank task: “The instrument’s early readings were (i) _____, so the team treated the apparent trend as (ii) _____ rather than established.” Choose one per column. Blank i: erratic / consistent / definitive. Blank ii: provisional / inevitable / irrelevant.

■ **Model and reasoning**

erratic; provisional. Unstable readings justify a tentative interpretation. Consistent and definitive oppose the reason for caution. Inevitable contradicts “rather than established”; irrelevant discards the trend instead of limiting confidence in it. The “so” connects the evidence quality to the status of the inference. Both choices must work in the completed passage.

Independent transfer

Original three-blank task: “Although the archive is (i) _____, its missing years do not make every conclusion (ii) _____: patterns repeated in the surviving independent records still justify a (iii) _____ claim.” Blank i: incomplete / comprehensive / fabricated. Blank ii: untenable / decisive / modest. Blank iii: qualified / universal / unrelated. Choose one per column and explain every rejection.

■ **Check after attempting**

incomplete; untenable; qualified. Missing years support incomplete, not comprehensive. Fabricated asserts falsification that the passage never supplies. The negation “do not make every conclusion” allows some justified inference: untenable means indefensible. Decisive would praise conclusions rather than name the criticism being rejected; modest is not the relevant failure. Repeated independent records support a limited claim, so qualified fits. Universal exceeds the incomplete evidence; unrelated breaks the stated support. The blanks are semantically dependent even though the answer columns are separate.

Common errors

- Choosing the first blank before reading the whole passage.
- Treating each column as a separate question.
- Missing a negation such as “do not” or “rather than”.
- Choosing a word of the right direction but the wrong strength.
- Not rereading the completed passage.

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Key terms

conjecture: A conclusion formed from incomplete evidence.

dogmatic: Holding an opinion firmly without considering evidence against it.

Check yourself

- Which blank should you fill first?
- "The data were (i) _____, so the conclusion remained (ii) _____." If (i) is "sparse", what is (ii)?
- Do you get partial credit for two correct blanks out of three?

Answers: the one with the strongest clue. Something like "tentative" or "provisional". No.

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Sentence equivalence: fitting context and matching complete meanings

GRE General Test

Sentence Equivalence: Fitting the Context and Matching Meanings

In 1852 the London doctor Peter Mark Roget, then aged 73, published his *Thesaurus of English Words and Phrases*. He had started the lists as a young man to help his own writing. Roget did not arrange words alphabetically but by idea, and he warned that words grouped together are not simply interchangeable: the right one depends on the sentence. GRE Sentence Equivalence tests exactly that warning. A pair of synonyms is not enough; both words must fit this sentence and give it the same meaning.

These questions ask you to:

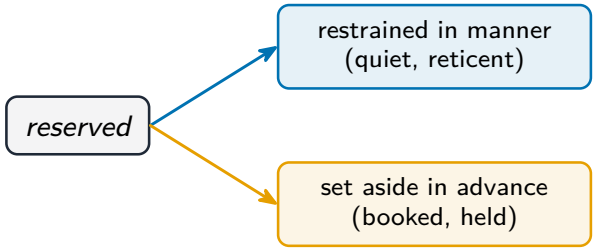
- Choose exactly two of six options to complete a sentence.
- Make sure both words fit the context's logic, tone and grammar.
- Make sure the two completed sentences mean the same, so that a **modest** 有限的 claim stays modest and an **evasive** 回避的 answer stays evasive.

Predict the meaning

- Read the sentence, find the clue, and say what the blank must mean before looking at the options.
- Clues include contrast words, causes, examples and restatements.
- Your prediction is a meaning, not a word.

Find two words that both fit

- Look for options that match your prediction. There should be two.
- A word may have several senses; only the sense this sentence needs counts.
- Watch for a tempting synonym pair that does not fit, and for a fitting word that has no partner.

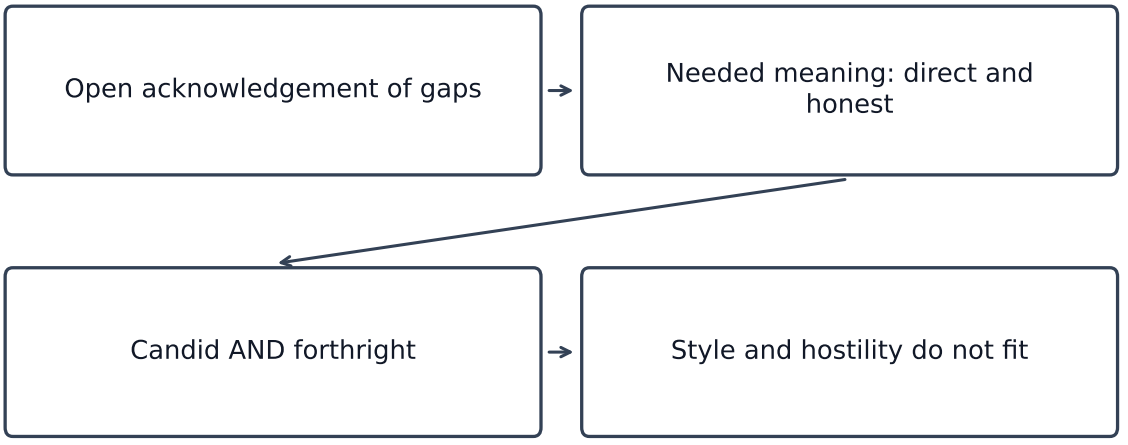


the complete sentence chooses one sense; a candidate synonym of the *other* sense is a trap

Match the sense the sentence needs, not just the dictionary word.

Check the whole sentence twice

- Read the sentence with each chosen word. Both must make the same complete statement.
- Check strength: "content" and "ecstatic" are both positive but not equivalent.
- No partial credit: both choices must be correct.



Both words must give the needed meaning: candid and forthright

Worked example: two senses of a word

"The usually _____ manager surprised everyone by speaking at length about her plans." Options include *reserved*, *reticent*, *booked*, *talkative*, *candid*, *held*.

- The contrast "surprised everyone by speaking at length" means she usually speaks little.
- *Reserved* (in the sense "quiet, holding back") and *reticent* both fit and give the same meaning.
- *Booked* and *held* match a different sense of "reserved" (set aside); *talkative* reverses the contrast.

Worked example: an original application

Select exactly two of six options: "The curator openly acknowledged gaps in the records; her account was therefore unusually _____ about what remained unknown." A. candid B. evasive C. ornate D. forthright E. repetitive F. hostile

Model and reasoning

A and D. Candid and forthright both express openness about uncertainty. Evasive reverses the meaning. Ornate describes style; repetitive describes recurrence; hostile adds an unsupported attitude. The sentence praises direct acknowledgement, not certainty about every detail.

Independent transfer

Select exactly two of six options: "Despite its imposing appearance, the device proved remarkably _____ to operate; first-time users completed the procedure without assistance." A. cumbersome B. intuitive C. unwieldy D. straightforward E. delicate F. novel

Check after attempting

B and D. Both make operation readily understandable in this context, supported by unaided new users. Cumbersome and unwieldy form a tempting negative pair but conflict with the evidence and contrast. Delicate concerns fragility or care, and novel concerns newness; neither matches ease of operation. Equivalent completed meanings matter more than choosing a dictionary synonym pair in isolation.

Common errors

- Choosing the first synonym pair you notice.
- Choosing a word in the wrong sense.
- Choosing two words of the same direction but different strength.
- Ignoring a contrast word.
- Choosing only one word, or three.

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Key terms

modest: Limited in size or degree; not exaggerated.

evasive: Avoiding a direct answer.

Check yourself

- How many options must you choose in sentence equivalence?
- Are "pleased" and "overjoyed" equivalent in "The team was _____ with a small improvement"?
- Why might a pair of synonyms be wrong?

Answers: exactly two. No: they differ in strength, and "small improvement" suggests a mild reaction. If neither fits the sentence's logic.

Reading comprehension: independent option tests and sentence selection

GRE General Test

Reading Comprehension: Testing Each Option and Selecting Sentences

In October 1950 the British mathematician Alan Turing published "Computing Machinery and Intelligence" in the journal *Mind*. He began by saying that the question "Can machines think?" was too vague to discuss, and proposed to replace it with a precise test, an "imitation game". Many later summaries said Turing claimed machines could think; in fact he predicted what people would say by the year 2000 and argued against objections. Reading exactly what a text claims, and no more, is the skill tested by GRE reading questions.

These questions ask you to:

- Test each answer option against the passage on its own.
- Keep the **scope** 范围 of the passage's claims.
- Recognise a **concession** 让步 and a stated limitation.
- Select the one sentence in a passage that does a stated job.

Three kinds of option

- Supported: the passage states it or it follows directly. Select it.
- Unsupported: the passage gives no evidence either way. Reject it.
- Contradicted: the passage says otherwise. Reject it.



watch scope: a study of volunteers at one site does not support "every person" or "every site"

Select only what the passage supports.

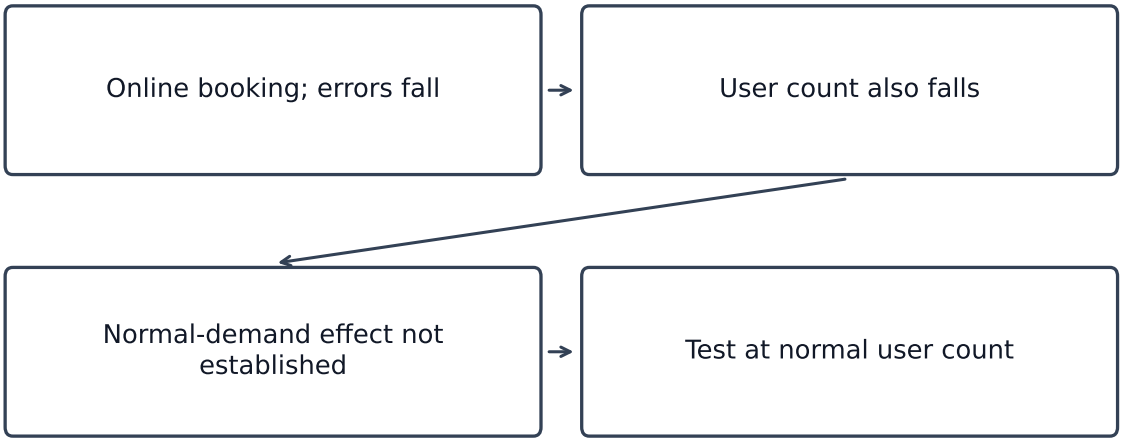
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Select one or more

- Treat each option as a true-or-false question about the passage.
- Any number of options may be correct; there is no partial credit.
- Do not choose an option because it sounds reasonable in real life.

Selecting a sentence

- Read the question's description carefully: "proposes a test", "limits the conclusion", "reports a rival explanation".
- Choose the whole sentence that does exactly that job, not a sentence near it.
- Concessions ("Although the new system may have helped…") often limit a claim.



Errors fell while the user count also fell, so test at normal demand

Worked example: testing options one by one

A passage says: "Volunteers at one site reported that a new app saved them time. Researchers have not yet tested it elsewhere."

- "Volunteers at one site found the app saved time." Supported.
- "The app saves time at every site." Unsupported: only one site was studied.
- "The app was tested at several sites." Contradicted.

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Worked example: an original application

Original passage: [1] A laboratory replaced handwritten booking with an online system. [2] Booking errors fell during a two-month trial. [3] The laboratory also reduced the number of users during that period. [4] Although the new system may have helped, the trial cannot establish its effect under normal demand. [5] Testing it with the usual number of users would address this limitation.
Select every supported statement: A. Errors fell during the trial. B. Lower demand may affect the interpretation. C. Online booking cannot reduce errors. Select the sentence that explicitly limits the generalisation.

■ **Model and reasoning**

A and B; sentence 4. A restates sentence 2. B follows from the changed user count and stated limitation. C turns lack of decisive evidence into evidence of impossibility. Sentence 3 reports the confounding change; sentence 4 states why the normal-demand conclusion remains limited. Select the whole sentence, not just its caution phrase.

Independent transfer

For that passage, identify the role of sentence 5 and explain whether a successful normal-demand trial would prove that every laboratory should adopt the system. Name one additional fact needed for that wider recommendation.

■ **Check after attempting**

Sentence 5 proposes a test addressing the specified limitation. A successful trial could support an effect in this laboratory under usual demand, but it would not establish a universal policy. Other laboratories may have different booking tasks, user access or costs. For example, evidence that the system remains usable for laboratories with limited internet access would inform transfer. The answer must connect the new fact to the proposed scope, not merely ask for unspecified "more research".

Common errors

- Choosing an option that is true in life but not supported by the passage.
- Widening the scope: one site becomes every site.
- Treating "not established" as "shown false".
- Choosing the sentence next to the right one.
- Stopping after the first correct option.

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Key terms

scope: How far a claim reaches: which group, place and time, and how strongly.

concession: Admitting a point before limiting or answering it.

Check yourself

- A passage says a drug "may reduce symptoms in some adults". Is "The drug cures adults" supported?
- In a select-one-or-more question, can all the options be correct?
- What does the sentence "Although the method is faster, it is less accurate" do?

Answers: no: it goes beyond "may" and "some". Yes. It concedes a benefit, then states a limit.

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Verbal arguments: assumptions, alternative causes and evidence direction

GRE General Test

Verbal Arguments: Assumptions, Alternative Causes and the Direction of Evidence

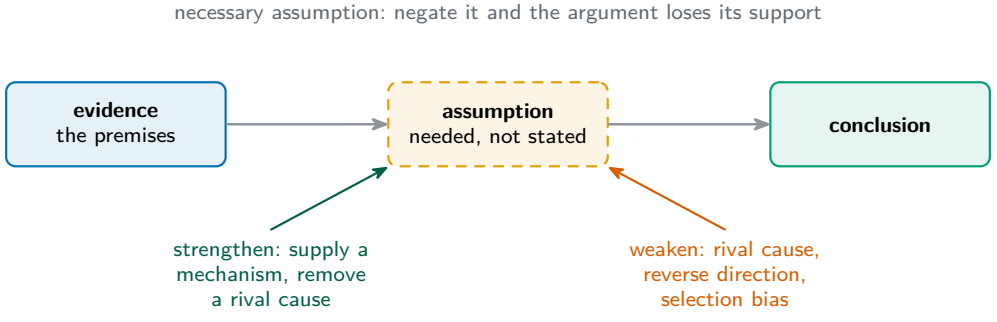
Between 1924 and 1932 researchers at the Hawthorne Works, a large Western Electric factory near Chicago, studied whether better lighting raised workers' output. Output rose when the lights were made brighter. It also rose in a comparison group whose lighting did not change, and in some tests it rose even when the lights were dimmed. The researchers concluded that something other than the light was at work, perhaps the attention the workers were getting. Later analysts have debated what the studies really showed. GRE argument questions ask exactly this: what is the argument assuming, and what other explanation could fit?

These questions ask you to:

- Find the evidence, the conclusion and the missing link between them.
- Identify a **necessary assumption** 必要假设: one the argument cannot work without.
- Recognise alternative causes, including **self-selection** 自我选择 into groups.
- Decide whether new information strengthens, weakens or does not affect an argument.

The structure of an argument

- Evidence: what is reported. Conclusion: what is claimed. Assumption: the unstated bridge.
- Causal arguments assume no rival cause, that the cause came first, and that the groups compared were alike.



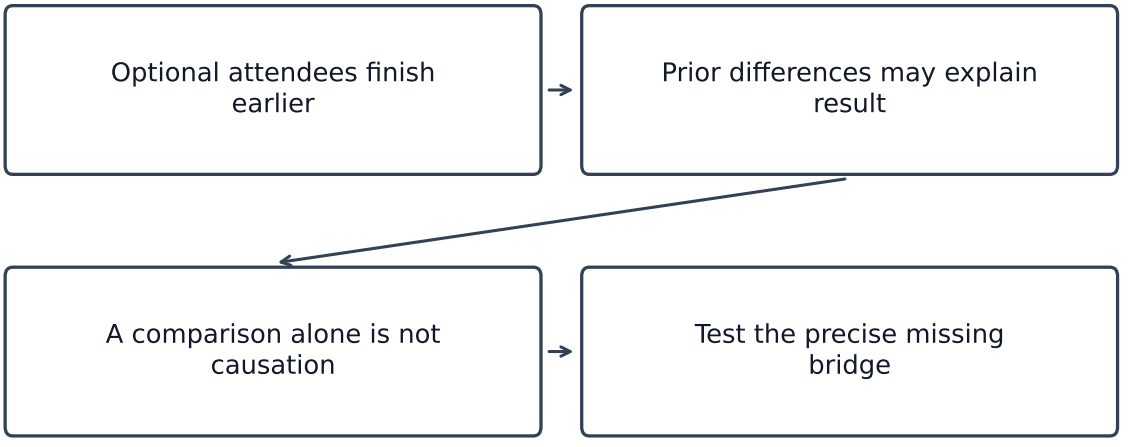
Strengthen and weaken by acting on the missing bridge.

Necessary assumptions

- A necessary assumption must be true for the argument to work.
- Negation test: write the opposite of the option. If the argument then collapses, the option is necessary.
- Do not confuse "necessary" with "sufficient": an assumption can be needed without proving the conclusion.

Strengthening and weakening

- Strengthen by removing a rival cause, adding a mechanism, or showing the comparison groups were alike.
- Weaken by naming a rival cause, reversing the direction of cause, or showing the groups differed beforehand.
- Information that is merely on the same topic, with no link to the conclusion, does neither.



Comparing self-selected groups is not causation; test the missing bridge

Worked example: a necessary assumption

"Students who used the new library app borrowed more books. So the app made students read more."

- Evidence: app users borrowed more. Conclusion: the app caused more reading.
- Necessary assumption: borrowing more books reflects reading more. Negate it ("extra borrowed books were not read"), and the conclusion fails.
- Rival cause by self-selection: keen readers may be the ones who downloaded the app. A comparison of borrowing before and after for the same students would strengthen the argument.

Worked example: an original application

Original argument: "Participants who chose an optional planning session finished their projects earlier. Therefore the session caused faster completion." Which most weakens the inference? A. Participants already completed previous projects earlier than nonparticipants. B. The room was recently painted. C. The session used printed notes. D. Many participants liked the session.

Model and reasoning

A. A pre-existing completion difference supplies a self-selection explanation. B and C have no stated connection to the outcome; D measures satisfaction rather than causal effect. A does not prove the session had no effect. It weakens the inference that the observed group difference establishes the claimed cause.

Independent transfer

An organiser argues: "A guide will prevent all registration errors because every applicant can read it." State a necessary assumption about accessibility, test its negation and distinguish it from the stronger claim that everyone actually reads the guide. Propose evidence that would strengthen the recommendation without proving "all".

Check after attempting

A necessary accessibility bridge is that no applicant is unable to understand the guide's relevant instructions. Negating it gives at least one applicant unable to understand them; the universal prevention argument loses its stated support. Accessibility is weaker than actual reading, and neither alone guarantees that every action is performed correctly. A comparison showing fewer errors among comparable applicants using the guide would strengthen an effectiveness claim, while still not proving zero errors for every applicant. Other necessary assumptions, such as instructions covering the possible errors, are valid when their negation is tied to the exact universal claim.

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Common errors

- Choosing an option that is on topic but has no link to the conclusion.
- Confusing a necessary assumption with one that would prove the conclusion.
- Missing self-selection when groups chose themselves.
- Treating a weakener as proof that the conclusion is false.
- Forgetting that cause could run the other way.

Key terms

necessary assumption: A premise that must be true for the argument to work.

self-selection: When people choose their own group, so the groups may differ from the start.

Check yourself

- "People who exercise sleep better, so exercise improves sleep." Name a rival explanation.
- How do you test whether an assumption is necessary?
- Does a weakener prove the conclusion false?

Answers: people who sleep well may have more energy to exercise (reverse cause), or another factor affects both. Negate it and see whether the argument collapses. No: it only reduces the support.

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Quantitative response formats: all valid values and numerical entry

GRE General Test

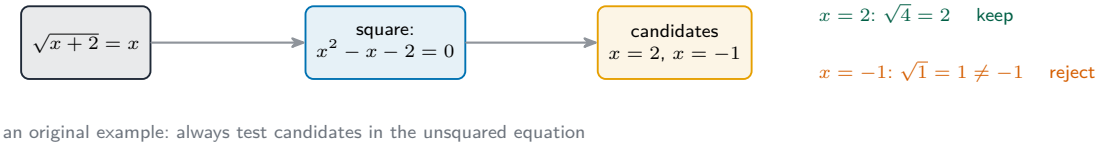
In about AD 250 Diophantus of Alexandria wrote *Arithmetica*, a collection of problems asking for the numbers that satisfy given conditions. Around 1637 the French lawyer Pierre de Fermat, reading a Latin translation, wrote in its margin that one such equation, $x^n + y^n = z^n$ for $n > 2$, has no whole-number solutions at all, and that he had a proof too long for the margin. It was finally proved by Andrew Wiles in 1994. Finding all the values that work, and only those, is the heart of two GRE response formats.

These questions ask you to:

- Answer "select one or more" questions by testing every option.
- Check each **candidate solution** 候选解 in the original conditions.
- Answer numerical-entry questions in the exact form asked, including **rounding** 舍入.

Select one or more answer choices

- Test each listed value against every condition.
- There may be one, several or all correct; there is no partial credit.
- A value that satisfies a simplified version may fail the original: check the original.



an original example: always test candidates in the unsquared equation

Squaring creates candidates; only the original equation decides.

Candidates from algebra

- Squaring, multiplying by an expression or cancelling can create or lose solutions.
- List the restrictions first: roots need non-negative arguments; denominators cannot be zero.
- Strict and inclusive inequalities differ at the endpoints.

Numerical entry

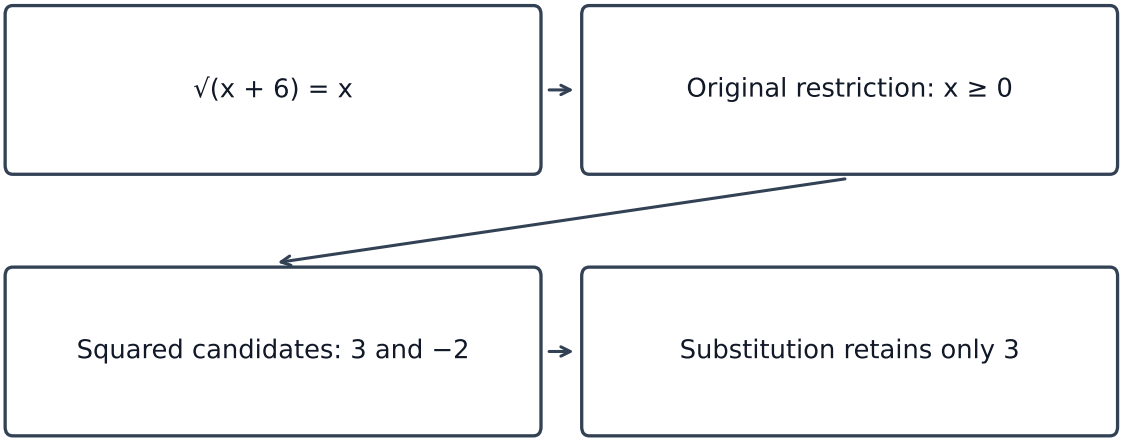
- One box: enter an integer or a decimal. Two boxes: enter a fraction (it need not be in lowest terms unless told).
- Enter the quantity asked for: a probability, not a percentage; a total, not a count of groups.
- Round only at the end, and only as instructed.



$P(\text{marked}) = 3/8$

0.375 ✓ | 37.5 (a percent) | 3 (the count)

Enter the form the box asks for.



Square both sides, then check each candidate in the original: only 3 survives

Worked example: all valid values

Select every integer n from 1 to 10 such that $n^2 - 5n + 6 \le 0$.

- Factor: $(n-2)(n-3) \le 0$, so $2 \le n \le 3$.
- Integers: 2 and 3. Check: $4 - 10 + 6 = 0$ and $9 - 15 + 6 = 0$, both ≤ 0.
- $n = 4$ gives $16 - 20 + 6 = 2 > 0$, so it fails.

Worked example: an original application

Select all real solutions of $\sqrt{x+6} = x$ from -3, -2, 0, 2, 3. Then enter the probability of drawing one marked counter from a bag with seven marked and thirteen unmarked counters, as a decimal.

■ **Model and reasoning**

3 only; 0.35. The original radical requires x nonnegative. Squaring gives $x^2 - x - 6 = (x-3)(x+2) = 0$. The candidate -2 fails both the sign condition and substitution. At 3 both sides equal three. For the probability, the denominator is twenty, so $P = 7/20 = 0.35$. Enter a probability, not the percent value 35.

Independent transfer

Select every listed integer satisfying $-2 \le x < 4$ and $x^2 > 1$: -3, -2, -1, 0, 1, 2, 3, 4. Then enter 7/12 as a decimal rounded to the nearest hundredth.

■ **Check after attempting**

-2, 2 and 3; 0.58. The first restriction keeps integers -2 through 3. The strict squared inequality excludes -1,0,1. Four fails the strict upper endpoint; -3 fails the lower bound. The decimal $7/12 = 0.58333\ldots$ rounds to 0.58. The rounding instruction here is explicit; an exact fraction response would require the fraction representation instead. Check each listed value and retain every valid one.

Common errors

- Stopping after one correct value.
- Keeping a candidate that fails the original equation.
- Entering 37.5 when the question asks for a probability of 0.375.
- Rounding before the last step.
- Including an endpoint excluded by a strict inequality.

Key terms

candidate solution: A value produced by a method that must still be checked in the original.

rounding: Replacing a number with a nearby simpler value to a stated precision.

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Check yourself

- Select all of -2, 0, 2 that satisfy $x^2 = 2x$.
- Enter $\frac{5}{8}$ as a decimal.
- Round $\frac{2}{3}$ to the nearest hundredth.

Answers: 0 and 2. 0.625. 0.67.

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Arithmetic: divisibility, remainders and multiplicative change

GRE General Test

In 1801 the 24-year-old Carl Friedrich Gauss published *Disquisitiones Arithmeticae* ("Arithmetical Investigations"). On its first pages he introduced the sign $\equiv$ for numbers that leave the same remainder: $17 \equiv 2 \pmod 5$, because both leave remainder 2 when divided by 5. With that notation, questions about remainders became algebra. GRE arithmetic questions use the same ideas in simple form: factors, multiples, remainders, and percentages that multiply.

These questions ask you to:

- Use **prime factorisation** 质因数分解 to find the **greatest common factor** 最大公因数 and the **least common multiple** 最小公倍数.
- Reason about the **remainder** 余数 when integers are divided.
- Work with ratios and with successive percentage changes.

Factors and multiples

- Write each number as a product of primes: $18 = 2 \cdot 3^2$, $24 = 2^3 \cdot 3$.
- GCF: take each shared prime to its smaller power: $2 \cdot 3 = 6$.
- LCM: take every prime to its larger power: $2^3 \cdot 3^2 = 72$.
- For two numbers, GCF $\times$ LCM = their product: $6 \times 72 = 432 = 18 \times 24$.

$$12 = \quad 2^2 \times 3^1$$

$$18 = \quad 2^1 \times 3^2$$

GCF: smaller powers $2^1 \times 3^1 = 6$

LCM: larger powers $2^2 \times 3^2 = 36$

check: $6 \times 36 = 216 = 12 \times 18$

Smaller powers for the GCF, larger powers for the LCM.

Remainders

- "n leaves remainder 2 when divided by 5" means $n = 5k + 2$ for some integer k.
- Work with the form: $2n = 10k + 4 = 5(2k) + 4$, remainder 4; $3n = 15k + 6 = 5(3k + 1) + 1$, remainder 1.
- A remainder is always at least 0 and less than the divisor.

- Even and odd: even ± even and odd ± odd are even; an odd times an odd is odd.

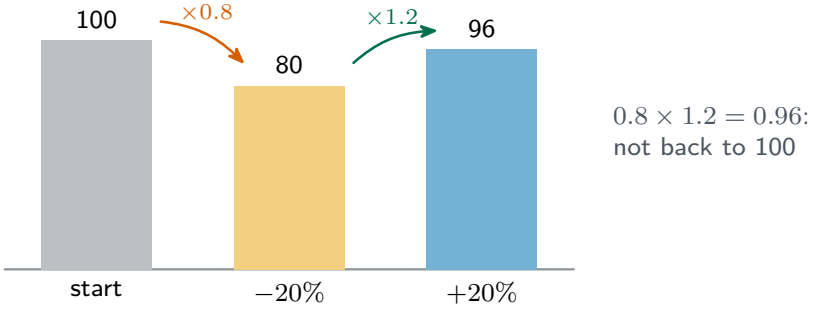
Ratios

- In a ratio $a : b$, the total has $a + b$ parts.
- Dividing 250 in the ratio 2 : 3: each of 5 parts is 50, so the shares are 100 and 150.

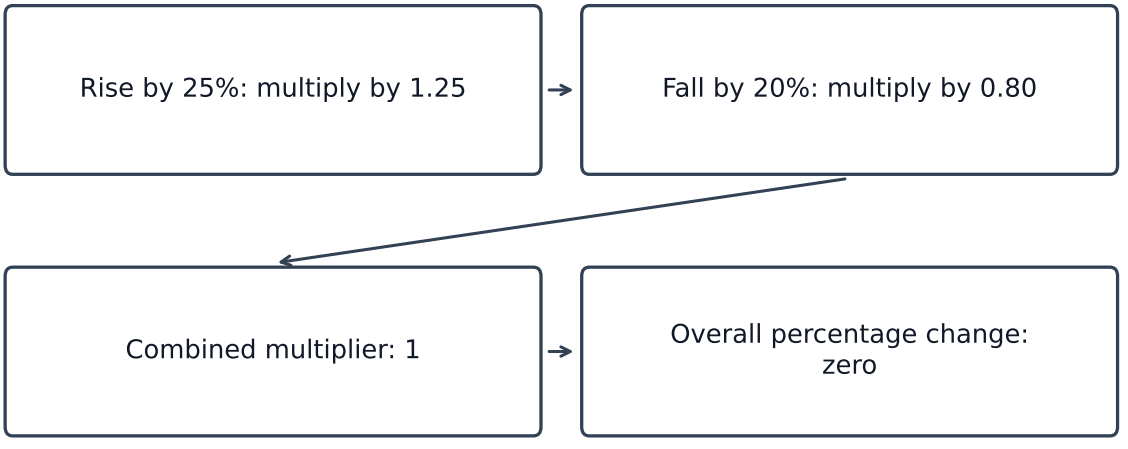
Multiplicative change

$$P_{\text{final}} = P_{\text{initial}}(1 + r_1)(1 + r_2)$$

- Successive percentage changes multiply; they do not add.
- A 20% fall followed by a 20% rise gives $0.8 \times 1.2 = 0.96$: a 4% fall overall.
- Each percentage is taken of a different base.



Successive percentages multiply.



×1.25 then ×0.80 gives ×1, so the overall change is zero

Worked example: factors and remainders

- $18 = 2 \cdot 3^2$ and $24 = 2^3 \cdot 3$: GCF 6, LCM 72.
- If $n = 5k + 2$, then $2n$ leaves remainder 4 and $n + 4 = 5k + 6 = 5(k + 1) + 1$ leaves remainder 1.
- Test with $n = 7$: $2n = 14 = 2 \times 5 + 4$, and $n + 4 = 11 = 2 \times 5 + 1$.

Worked example: an original application

Find the greatest common factor and least common multiple of 36 and 60. A price rises 25% then falls 20%. Find its overall percentage change.

Model and reasoning

$36 = 2^2 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$. Shared smaller powers give GCF=12; larger required powers give LCM=180. Their product $12 \cdot 180 = 36 \cdot 60$ checks the result. The price multiplier is $1.25 \cdot 0.80 = 1$, so the overall change is zero percent. The two changes use different reference prices; adding +25 and -20 would give a wrong five-percent increase.

Independent transfer

An integer n has remainder 5 on division by 8. Find the remainder of $3n+7$ on division by 8, including for negative n. A value rises 10% in one year and 20% the next. What single percentage reduction restores its original value?

Check after attempting

Write $n = 8k + 5$ for integer k. Then $3n + 7 = 24k + 22 = 8(3k + 2) + 6$, so the remainder is six for all integer k. The final value is 1.32 times the original. Restoring it multiplies by $1/1.32 = 25/33$, a reduction of $1 - 25/33 = 8/33$, or about 24.24%. A 30% reduction does not reverse compounded growth. The remainder convention uses $0 \leq r < 8$ even when n is negative.

Common errors

- Taking the larger powers for the GCF.
- Giving a remainder equal to or greater than the divisor.
- Adding successive percentages.
- Assuming a fall and an equal rise cancel out.
- Treating a part of a ratio as a fraction of the other part.

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Key terms

prime factorisation: Writing a number as a product of primes.

greatest common factor: The largest integer that divides each of the numbers.

least common multiple: The smallest positive integer that is a multiple of each of the numbers.

remainder: What is left over after division by a whole number of times.

Check yourself

- Find the GCF of 28 and 42.
- n leaves remainder 3 when divided by 7. What remainder does $n + 5$ leave?
- A price rises 10% and then rises 10% again. What is the overall percentage increase?

Answers: 14. $n + 5 = 7k + 8 = 7(k + 1) + 1$: remainder 1. $1.1^2 = 1.21$: 21%.

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Algebra: domains, systems, inequalities and functions

GRE General Test

In 1591 the French lawyer François Viète, who also broke Spanish codes for King Henri IV, published *In artem analyticem isagoge* ("Introduction to the Analytic Art"). He used letters for quantities in equations: vowels for unknowns and consonants for known values. For the first time a whole family of problems could be solved at once, in general. GRE algebra asks you to work with that generality carefully: where an expression is defined, which solutions survive, and how parameters change a system.

These questions ask you to:

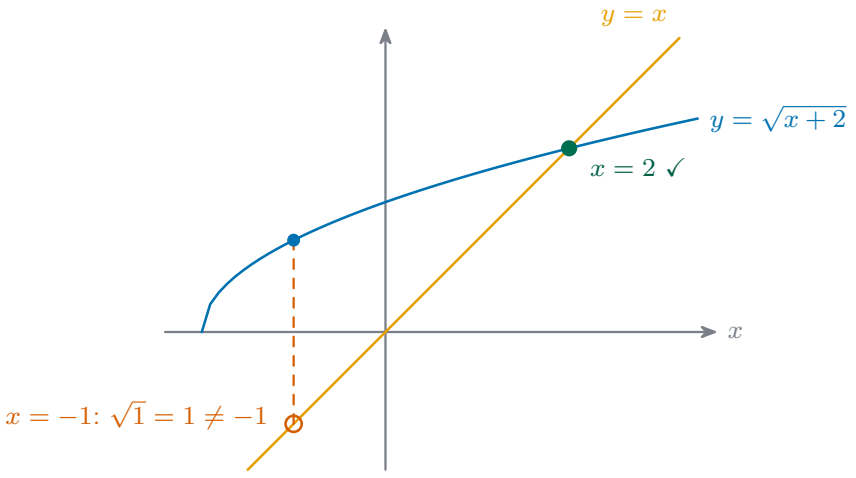
- State each **domain restriction** 定义域限制 before solving.
- Reject an **extraneous root** 增根 created by squaring or by multiplying through.
- Solve inequalities with a **sign chart** 符号表.
- Find the **simultaneous solution** 联立解 of a system, and work with functions.

Domains first

- A denominator cannot be zero; an even root needs a non-negative inside.
- Write the restriction before you simplify: $\dfrac{x^2-4}{x-2}$ requires $x \neq 2$, even though it simplifies to $x+2$.
- A solution that breaks a restriction must be rejected.

Equations that create false roots

- Squaring both sides can add a solution: check every candidate in the original.
- For $\sqrt{x+2} = x$: we need $x \geq 0$; squaring gives $x^2 - x - 2 = 0$, candidates 2 and -1; only 2 works.



Squaring can add a false root: check in the original.

Inequalities

- Move everything to one side and factor.
- Mark the roots on a number line and test one value in each interval.
- Include the roots for $\leq$ or $\geq$, exclude them for $<$ or $>$.
- Multiplying or dividing by a negative number reverses the sign.

$(x-1)(x-4) < 0$: test one value in each interval

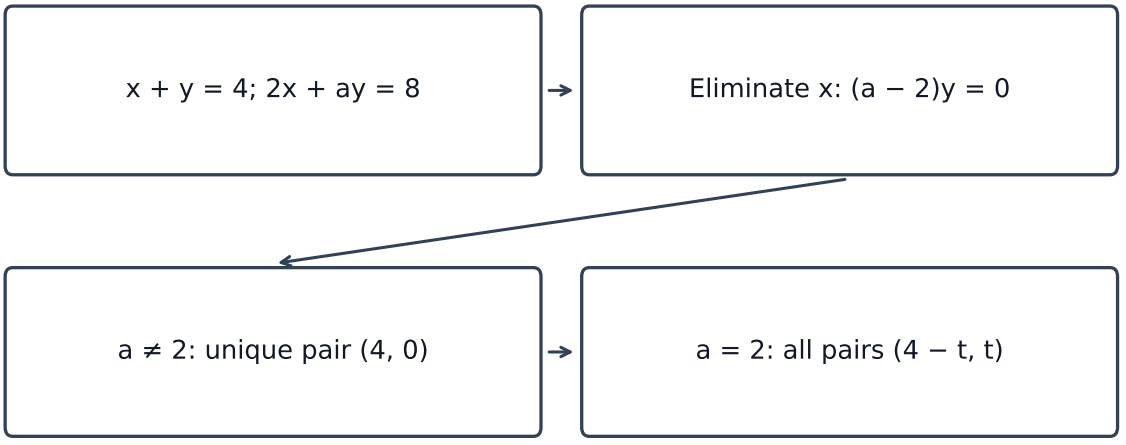


$1 < x < 4$, open circles: equality not allowed

Order the roots, test each interval, include endpoints only when equality is allowed.

Systems and functions

- Solve linear systems by elimination or substitution, and check in both equations.
- If one equation is a multiple of the other, there are infinitely many solutions; if only the left sides match, none.
- $f(g(x))$: apply g first, then f.



The value of a decides between one solution and infinitely many

Worked example: four short problems

- $\sqrt{x+2} = x$: only $x = 2$ (check: $\sqrt{4} = 2$; -1 gives $\sqrt{1} = 1 \neq -1$).
- $(x-1)(x-4) \leq 0$: the parabola opens upwards and is at or below zero between its roots, so $1 \leq x \leq 4$.
- $x + y = 10$ and $x - y = 2$: adding gives $2x = 12$, so $x = 6$, $y = 4$.
- $f(t) = 2t + 1$, $g(x) = x^2$: $f(g(3)) = f(9) = 19$, but $g(f(3)) = g(7) = 49$.

Worked example: an original application

Solve $(x^2 - 4)/(x - 2) = 5$ on its original real domain. Solve $(x+1)(x-3) \leq 0$ and find $f(g(2))$ for $f(t) = t^2$, $g(x) = x - 3$.

Model and reasoning

The rational equation excludes x=2. For allowed x it reduces to x+2=5, giving x=3, which substitutes correctly. The quadratic is nonpositive between its roots, including them: $-1 \leq x \leq 3$. The composition applies g first: $g(2) = -1$ and $f(g(2)) = 1$. Cancelling a factor does not restore excluded inputs.

Independent transfer

For real a, classify the system $x + y = 4$, $2x + ay = 8$ as having one, none or infinitely many solutions. Compare with the second equation $2x + ay = 9$. Give solutions in every consistent case.

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Check after attempting

Subtract twice the first equation. In the first system $(a-2)y=0$: for a not equal to two, y=0,x=4, uniquely. For a=2, both equations are the same relation, giving every pair $(4-t,t)$. There is no inconsistent case in this first system. With right side nine, $(a-2)y=1$. For a not equal to two, $y=1/(a-2)$ and $x=4-1/(a-2)$, uniquely. For a=2 it demands 0=1, so no solution. A zero coefficient row means inconsistency only when the right side is nonzero.

Common errors

- Simplifying before noting the restrictions.
- Keeping a candidate that fails the original equation.
- Forgetting to reverse an inequality when dividing by a negative.
- Including endpoints in a strict inequality.
- Applying the functions in $f(g(x))$ in the wrong order.

Key terms

domain restriction: A value the variable cannot take.

extraneous root: A candidate produced by the method that does not satisfy the original equation.

sign chart: A number line showing where an expression is positive or negative.

simultaneous solution: Values that satisfy all the equations of a system at once.

Check yourself

- Solve $|x-3|<2$.
- For which a do $x+y=4$ and $2x+ay=8$ have infinitely many solutions?
- State the domain of $\dfrac{1}{x-5}$.

Answers: $1<x<5$. $a=2$. All real x except 5.

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Geometry: labelled facts, coordinates, circles and solids

GRE General Test

In 1899 the German mathematician David Hilbert published *Grundlagen der Geometrie* ("Foundations of Geometry"). He rebuilt Euclid's geometry so that every result followed from stated axioms, with no step resting on what a picture seemed to show. He is said to have remarked that one should be able to say "tables, chairs and beer mugs" instead of "points, lines and planes", and the proofs should still work. GRE geometry figures are "not necessarily drawn to scale" for the same reason: the given facts decide, not the picture.

These questions ask you to:

- Reason from labelled facts, not from how a figure looks.
- Use right triangles, coordinates, the distance formula and the **midpoint** 中点.
- Find arc length and the area of a **sector** 扇形.
- Use a **scale factor** 缩放比例 for lengths, areas and volumes.

Use the given facts

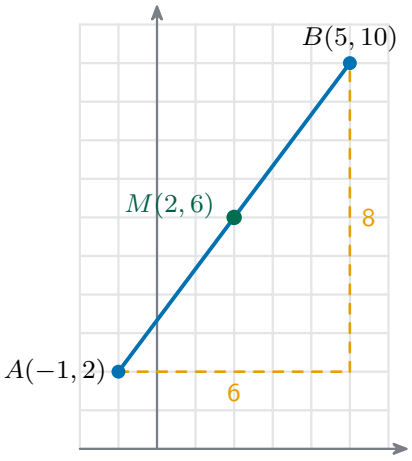
- Unless the question says the figure is to scale, do not measure it or trust apparent right angles, equal lengths or parallel lines.
- Use what is labelled: lengths, angles, "parallel", "perpendicular", "tangent", "centre".
- Points shown on a straight line are on that line, and the order of points shown can be relied on.

Triangles and coordinates

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M = \left(\tfrac{x_1 + x_2}{2}, \tfrac{y_1 + y_2}{2}\right)$$

- The distance formula is Pythagoras applied to the horizontal and vertical legs.
- Common right triangles: 3–4–5, 5–12–13, 6–8–10, and 45°–45°–90°, 30°–60°–90°.

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$d = \sqrt{6^2 + 8^2} = 10$
M averages the coordinates

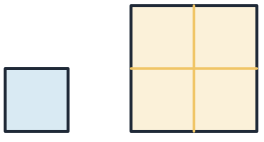
Distance from a right triangle; midpoint by averaging.

Circles and sectors

- Circumference $2\pi r$; area πr^2.
- A sector with central angle $\theta°$ is $\dfrac{\theta}{360}$ of the circle: arc $\dfrac{\theta}{360} \cdot 2\pi r$, area $\dfrac{\theta}{360} \cdot \pi r^2$.
- A tangent is perpendicular to the radius at the point of contact.

Solids and scaling

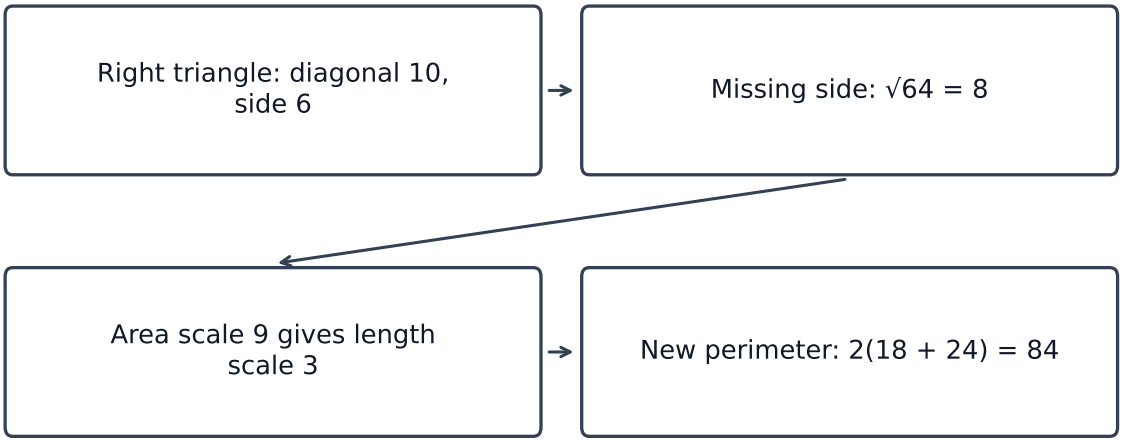
- Rectangular box $V = lwh$; cylinder $V = \pi r^2 h$, surface area $2\pi r^2 + 2\pi rh$.
- If every length is multiplied by k: areas by k^2, volumes by k^3.
- Work backwards too: an area scale of 9 means a length scale of 3.



length: $\times k = \times 2$
area: $\times k^2 = \times 4$ (four unit squares)
volume: $\times k^3 = \times 8$

given an area ratio of 4, take the square root: the length ratio is 2

Length k, area k squared, volume k cubed.



Area scale 9 means length scale 3: the 6 by 8 rectangle becomes 18 by 24

Worked example: coordinates, a sector and scaling

- $A(-1,2)$ and $B(5,10)$: the legs are 6 and 8, so $d=10$; midpoint $(2,6)$.
- A circle of radius 6 with a 60° sector: area $\frac{60}{360}\cdot 36\pi = 6\pi$; arc $\frac{60}{360}\cdot 12\pi = 2\pi$.
- Enlarging every length by 3 multiplies area by 9 and volume by 27, not by 3.

Worked example: an original application

Points A=(-1,2) and B=(5,10) are given. Find their distance and midpoint. A right circular cylinder has radius 3 cm and height 4 cm. Find its volume and total surface area, including both ends.

Model and reasoning

Coordinate differences are six and eight. Distance is $\sqrt{6^2+8^2}=10$ and midpoint $((-1+5)/2,(2+10)/2)=(2,6)$. Cylinder volume is $V=\pi r^2 h=\pi(3\text{ cm})^2(4\text{ cm})=36\pi\text{ cm}^3$. Total area is $S=2\pi r^2+2\pi rh=2\pi(3\text{ cm})^2+2\pi(3\text{ cm})(4\text{ cm})=42\pi\text{ cm}^2$. Volume and surface area measure different quantities.

Independent transfer

A rectangle has diagonal 10 cm and one side 6 cm. A similar rectangle has area nine times as large. Find its perimeter. Explain which labelled facts permit Pythagoras, and whether the same calculation would hold for an arbitrary quadrilateral.

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■ **Check after attempting**

A rectangle's adjacent sides are perpendicular. Its diagonal forms a right triangle, so the other side is $b = \sqrt{d^2 - a^2} = \sqrt{(10\text{ cm})^2 - (6\text{ cm})^2} = 8$ cm. Area scale nine means length scale three. The new sides are 18 and 24 cm, giving perimeter $P = 2(a + b) = 2(18 + 24)$ cm $= 84$ cm. An arbitrary quadrilateral need not supply a right angle, so its drawing alone would not justify the equation.

Common errors

- Trusting a figure that is not to scale.
- Forgetting the $\frac{\theta}{360}$ fraction for a sector.
- Multiplying area or volume by k instead of k^2 or k^3.
- Leaving out one end when finding a cylinder's total surface area.
- Mixing up radius and diameter.

Key terms

midpoint: The point halfway between two points.

sector: The part of a circle between two radii and an arc.

scale factor: The number every length is multiplied by in an enlargement.

Check yourself

- Find the circumference of a circle with radius 5.
- Find the area of a 90° sector of a circle with radius 4.
- A cube's side changes from 2 to 6. By what factor does its volume change?

Answers: 10π. $\frac{1}{4} \cdot 16\pi = 4\pi$. 27.

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Data analysis: spread, weighted averages and conditional probability

GRE General Test

In the autumn of 1973 the University of California, Berkeley, admitted about 44% of the men who applied to its graduate programmes but only about 35% of the women. It looked like clear bias. When the statistician Peter Bickel and two colleagues examined the figures department by department, published in *Science* in 1975, they found that women were admitted at similar or higher rates in most departments. Women had applied more often to the departments that admitted few applicants of any kind. Combining groups of different sizes can reverse a pattern, an effect now called Simpson's paradox. GRE data questions test the same care with totals, weights and denominators.

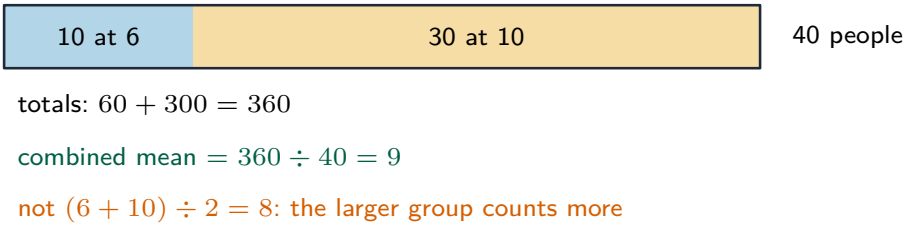
These questions ask you to:

- Combine groups using a **weighted average** 加权平均.
- Compare mean, median, range and **standard deviation** 标准差.
- Find a **conditional probability** 条件概率, including draws without replacement.
- Use "or" correctly, including for **mutually exclusive** 互斥的 events.

Weighted averages

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

- Combine totals, not means. The larger group counts more.
- The combined mean always lies between the group means, nearer the larger group.



Combine totals, not means: the larger group counts more.

Centre and spread

- An outlier pulls the mean but hardly moves the median.
- Adding a constant to every value shifts the mean and median but leaves the range and standard deviation unchanged.

- Multiplying every value by $k > 0$ multiplies the mean, median, range and standard deviation by k.

Conditional probability and draws

- "Given that" restricts the group: divide by the size of that group.
- Without replacement, the second probability uses the new totals: $\frac{4}{10} \cdot \frac{3}{9}$, not $\left(\frac{4}{10}\right)^2$.

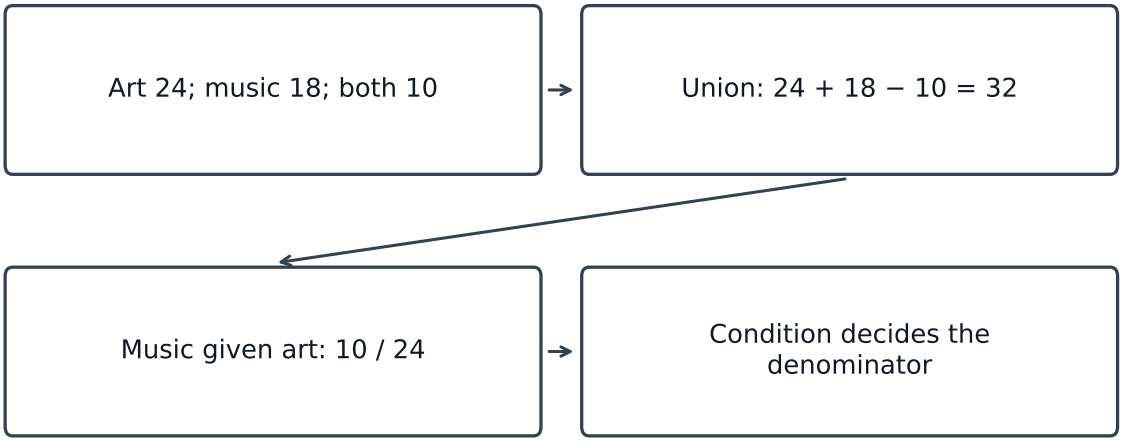


the 4 marked counters are the new denominator

Condition on the marked counters.

"Or" and overlaps

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
- If A and B are mutually exclusive, they cannot both happen, so $P(A \text{ or } B) = P(A) + P(B)$.
- For counts: union = group 1 + group 2 − both.



For music given art, divide by the 24 art students, not by the union of 32

Worked example: a weighted mean and an outlier

- 10 people with mean 6 and 30 people with mean 10: totals $60 + 300 = 360$, so the mean is $\frac{360}{40} = 9$, not the unweighted 8.
- Data 1, 2, 3, 4, 20: mean $\frac{30}{5} = 6$, median 3, range 19. The outlier 20 raises the mean above every other value.
- Adding 5 to each value: mean 11, median 8, standard deviation unchanged.

Worked example: an original application

Group A has 12 observations with mean 5; group B has eight with mean 10. Find the combined mean. For the data 1,2,3,4,20 find mean, median and range. Describe how adding 7 to every value changes the standard deviation.

Model and reasoning

The combined total is $12(5) + 8(10) = 140$ across twenty observations, so mean is seven. An unweighted average of five and ten would incorrectly ignore group sizes. In the second data set, sum is thirty, mean six, median three and range nineteen. Adding seven shifts all values and their mean together; every deviation from the mean is unchanged, so standard deviation is unchanged.

Independent transfer

Of 40 students, 24 study art, 18 study music and 10 study both. Find the probability of music given art and the probability of at least one subject. Are the events independent? Then explain what happens to standard deviation if all measurements are multiplied by -3.

Check after attempting

Conditioning on art gives $P(M \mid A) = 10/24 = 5/12$. The union count is $24 + 18 - 10 = 32$, so union probability is $4/5$. Independence would require $P(A \cap M) = P(A)P(M)$, but $10/40 = 1/4$ differs from $(24/40)(18/40) = 27/100$. The events are not independent. Scaling every deviation by -3 scales its square by nine and the square root by three, so standard deviation triples. It cannot become negative.

Common errors

- Averaging the group means without weighting.
- Saying that adding a constant changes the spread.
- Dividing by the whole population in a “given” question.
- Squaring the first probability for draws without replacement.
- Counting the overlap twice in a union.

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Key terms

weighted average: A mean in which each group counts in proportion to its size.
standard deviation: A measure of how far values typically lie from the mean.
conditional probability: The probability of an event within a restricted group.
mutually exclusive: Unable to happen at the same time.

Check yourself

- Group A has 5 people with mean 8; group B has 15 with mean 4. Find the combined mean.
- 24 students take art, 18 take music and 10 take both. Find $P(\text{music} \mid \text{art})$.
- A and B are mutually exclusive, with $P(A) = 0.3$ and $P(B) = 0.4$. Find $P(A \text{ or } B)$.

Answers: $\frac{40+60}{20} = 5$. $\frac{10}{24} = \frac{5}{12}$. 0.7.

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Analyze an Issue: instruction families and a reasoned qualified position

GRE General Test

Analyze an Issue: Instruction Families and a Qualified Position

Between October 1787 and May 1788, three men writing as "Publius", Alexander Hamilton, James Madison and John Jay, published 85 essays in New York newspapers urging voters to accept the new United States Constitution. Many essays began by stating the opponents' best objection. In *Federalist* No. 10 Madison took on the common view that a republic must be small, and argued instead that a large one would control factions better. The essays are still read because they answer the strongest case against them. The GRE Issue task asks for a small version of that work, in 30 minutes, under a specific instruction.

These questions ask you to:

- Recognise which instruction comes with the claim, and do what it asks.
- Write a **qualified thesis** 有条件的论点 that states when the claim holds.
- Weigh a **policy consequence** 政策后果 where the instruction asks.
- Use a **counterexample** 反例 or objection, and answer it.

The instruction families

The claim always comes with one instruction. The main families ask you to:

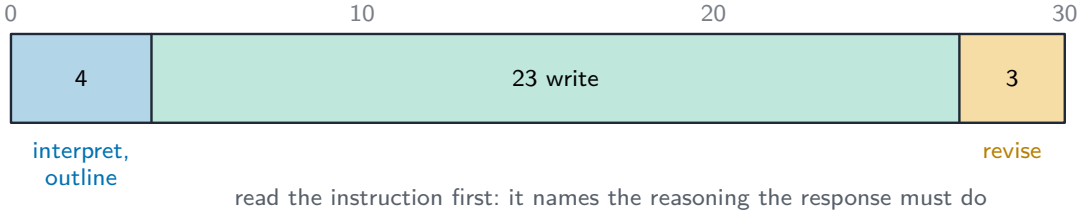
- Discuss how far you agree, and how the claim might or might not hold.
- Address the most compelling reasons or examples against your position.
- Evaluate a claim together with the reason given for it.
- Discuss the consequences of a recommendation or policy.
- Discuss two opposing views and which you find more convincing.

Underline the instruction before planning.

A qualified thesis

- State your position with its scope: "This holds for X, because…, but not for Y, because…".
- If the instruction includes a reason, judge the reason separately from the claim: a good claim can have a weak reason.
- Keep the thesis to one or two sentences.

Planning the 30 minutes

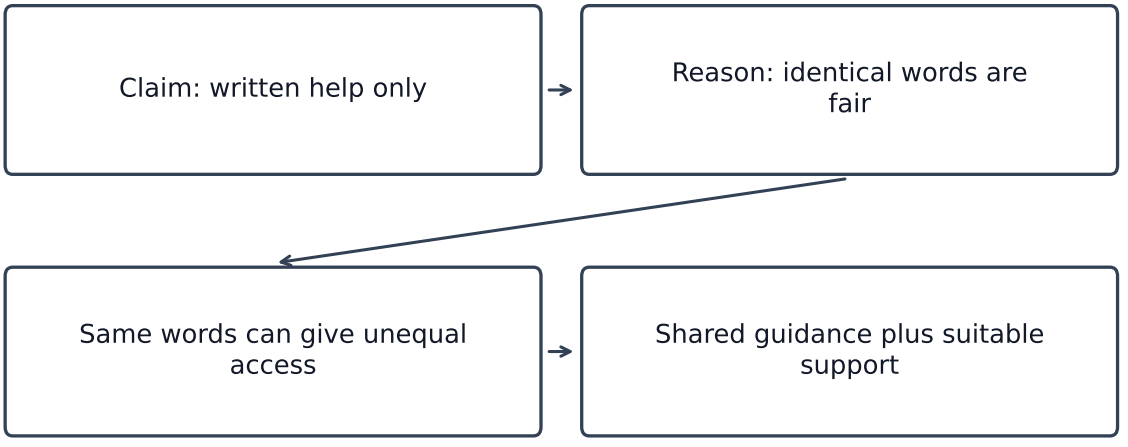


A suggested practice budget, not a required paragraph count.

- A practice budget: about 4 minutes to read and outline, 23 to write, 3 to revise.
- Outline: thesis; two or three developed reasons; the strongest objection and your answer; conclusion.

Developing and answering

- Explain each example: what happens, and why it supports your point.
- For a policy, consider who benefits, who bears the cost, and what could go wrong.
- A counterexample shows where the claim fails; use it to set the limit of your thesis.



Identical help is not equal access: pair shared guidance with suitable support

Worked example: a claim and its reason

Claim: “Universities should let every student replace a laboratory requirement with independent reading.” Reason: “All meaningful learning comes from books.”

- The instruction asks you to judge both. The reason is too strong: it ignores learning from direct measurement, mistakes in procedure and working with others.
- A qualified thesis: reject universal replacement, but accept reading as preparation, and allow a remote or simulated alternative where a laboratory is impossible.
- Counterexample to the reason: a student who has read about titration still has to learn to judge an end point by eye.

Worked example: an original application

Original Issue prompt. Claim: “Public learning centres should provide help only through written guides.” Reason: “Giving every visitor identical information is the fairest policy.” Discuss how far you agree with both the claim and its reason. Explain circumstances and consequences that affect your position.

Model and reasoning

Written guides can make public services clearer and more consistent, but they should not be the only source of help in a learning centre. I also question the reason offered for that policy. Fairness requires dependable information, yet giving everyone identical words does not ensure that everyone can use them. A better policy combines shared written guidance with appropriate personal support.

The argument has a real strength. Visitors should not receive contradictory instructions simply because different staff members answer their questions. A clear guide can state opening times, booking rules and available services in a form that visitors can consult again. In a hypothetical centre where staff previously gave different answers about course registration, a common written procedure could reduce confusion. It could also free staff from repeating routine explanations and allow them to spend more time on difficult questions. These are practical reasons to invest in guides.

However, consistency of information is different from equality of access. A visitor with limited reading ability may be unable to follow the guide. Another may understand each sentence but not know which rule applies to an unusual case. If both receive only the same document as a confident regular visitor, the centre has treated them identically while leaving them with different opportunities to learn. Personal explanation can remove that barrier without changing the underlying rule. Fairness therefore supports consistent decisions and suitable assistance, rather than one communication method for all.

The proposed restriction may also produce costs that its reason ignores. Suppose a visitor misunderstands a booking instruction and repeatedly submits an incomplete

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form. A short conversation might identify the difficulty immediately. Refusing that conversation can increase delay for the visitor and create extra correction work for staff. The centre should compare the full process, not just the time saved at the first enquiry. This example is hypothetical, but it explains a mechanism through which a seemingly efficient policy could waste resources.

There are circumstances in which written-only support may be reasonable for a limited service. A simple self-service room used by experienced members might need only clear notices and an accessible guide. Even then, the centre should provide a way to report an exceptional problem and check whether users actually understand the instructions. A limited case does not justify extending the restriction to every service and every visitor.

I would therefore retain written guides as the common reference and allow staff to explain and apply them. Staff should record recurring difficulties so the guide improves, and unusual decisions should remain accountable to shared rules. This approach accepts the value of identical core information while rejecting the claim that identical communication is always fair. It preserves consistency without making a visitor's ability to interpret a document the hidden condition for receiving help.

Independent transfer

Compare three instruction families using this issue: (a) assess when a recommendation works, (b) discuss both a claim and its stated reason, (c) address the strongest challenge to your own position. Write a different planning sentence for each. Explain why a fixed agree/disagree template could miss the task.

■ Check after attempting

(a) "Written-only help may work for simple routine services used by experienced visitors, but fails when accessibility or unusual cases demand explanation." (b) "I reject exclusive written help and distinguish the reason's valid concern for consistency from its unsupported equation of identical words with fairness." (c) "The strongest challenge is that personal support may introduce inconsistent advice; shared rules, records and staff training can reduce that risk." Other plans are valid if they perform the requested action. A generic position without conditions, a judgement on the reason or an answered challenge can omit the instruction even when its prose is fluent. Use human review for reasoning and language; local checks do not assign an official writing score.

Common errors

- Writing a general essay that ignores the instruction.
- Judging the claim but not the reason when both are required.
- A thesis with no limits, defended only by examples that fit.
- Naming consequences without weighing them.

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- Running out of time for the objection and conclusion.

Key terms

qualified thesis: A position that states the conditions under which a claim holds.

policy consequence: A likely result of carrying out a recommendation.

counterexample: A case that shows a general claim is not always true.

Check yourself

- The instruction says to "discuss the consequences of following the recommendation". What must your essay include?
- A claim seems right but its reason is weak. What can your thesis say?
- In a 30-minute task, roughly how long should you spend outlining?

Answers: likely results, good and bad, and how they affect your position. That the claim may hold for other reasons, while the given reason fails. About 4 minutes.

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