Geometry: labelled facts, coordinates, circles and solids

GRE General Test

Scope and task

Original classroom practice within the reviewed shorter-test task types. This does not simulate an adaptive test or predict a scaled or writing score.

- Use given relationships rather than a diagram's apparent scale
- Connect coordinate distances, slopes and geometric properties
- Distinguish linear, area and volume scale factors

sector : A circle region bounded by two radii and their included arc.

scale factor : The multiplier relating corresponding lengths of similar figures.

Read the evidence and choose a method

Extract the facts a diagram supplies: lengths, right-angle marks, parallel lines, circle radii and stated shape properties. Avoid measuring the screen. Angle sums and similarity give relationships when their conditions hold. Pythagorean reasoning needs a right triangle; it does not apply to every drawn triangle.

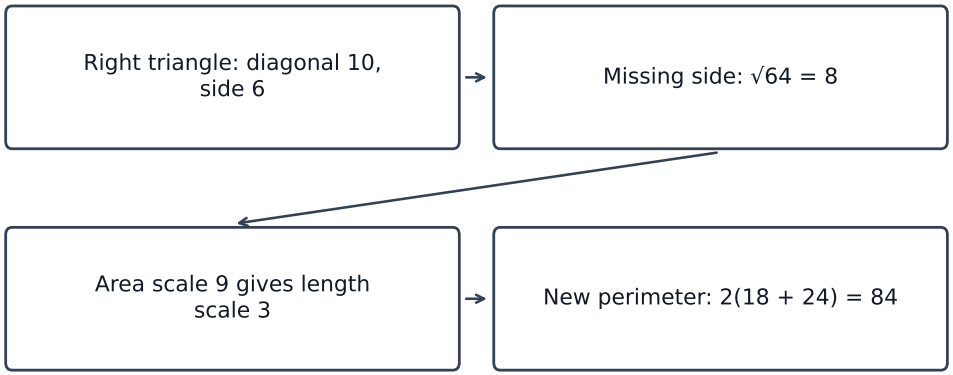
For two coordinates, use midpoint $((x_1+x_2)/2,(y_1+y_2)/2)$ and distance $\sqrt{((x_2-x_1)^2+(y_2-y_1)^2)}$. Slope is $\Delta y/\Delta x$ when $\Delta x \neq 0$. A vertical line has no finite slope. Combine coordinate and shape facts where useful; perpendicular nonvertical lines have slopes whose product is -1.

A circle has circumference $2\pi r$ and area πr^2. An arc or sector uses its fraction of a full turn. Rectangular solids use volume lwh; cylinders use $\pi r^2 h$. Surface area counts exposed faces or surfaces, not the enclosed volume. Attach units so an area answer is not confused with a length or volume.

For similar figures with linear factor k, corresponding areas multiply by k^2 and volumes by k^3. If a question provides only an area ratio, take a square root to recover the length ratio; do not copy the area ratio directly. General-test geometry uses elementary relationships, with no trigonometry or calculus required for this lesson's tasks.

Worked reasoning

Points A$(-1,2)$ and B$(5,10)$ have $\Delta x=6$, $\Delta y=8$. Distance $d=\sqrt{(6^2+8^2)}=10$ and midpoint $M=((-1+5)/2,(2+10)/2)=(2,6)$. For a circle of radius 6, a 60° sector is 1/6 of a full circle: area$=(60/360)\pi 6^2=6\pi$; arc length$=(60/360)2\pi 6=2\pi$. Enlarging every length by 3 multiplies area by 9 and volume by 27, not 3.



Area scale 9 means length scale 3: the 6 by 8 rectangle becomes 18 by 24

Conditions and common errors

Use labelled facts, include required π or units, and distinguish a perimeter from an area or a sector from its arc.

Original application

Points A=(-1,2) and B=(5,10) are given. Find their distance and midpoint. A right circular cylinder has radius 3 cm and height 4 cm. Find its volume and total surface area, including both ends.

Model and reasoning

Coordinate differences are six and eight. Distance is $\sqrt{6^2+8^2}=10$ and midpoint $((-1+5)/2,(2+10)/2)=(2,6)$. Cylinder volume is $V=\pi r^2 h=\pi(3\text{ cm})^2(4\text{ cm})=36\pi\text{ cm}^3$. Total area is $S=2\pi r^2+2\pi rh=2\pi(3\text{ cm})^2+2\pi(3\text{ cm})(4\text{ cm})=42\pi\text{ cm}^2$. Volume and surface area measure different quantities.

Independent transfer

A rectangle has diagonal 10 cm and one side 6 cm. A similar rectangle has area nine times as large. Find its perimeter. Explain which labelled facts permit Pythagoras, and whether the same calculation would hold for an arbitrary quadrilateral.

■ **Check after attempting**

A rectangle's adjacent sides are perpendicular. Its diagonal forms a right triangle, so the other side is $b = \sqrt{d^2 - a^2} = \sqrt{(10\text{ cm})^2 - (6\text{ cm})^2} = 8$ cm. Area scale nine means length scale three. The new sides are 18 and 24 cm, giving perimeter $P = 2(a + b) = 2(18 + 24)$ cm $= 84$ cm. An arbitrary quadrilateral need not supply a right angle, so its drawing alone would not justify the equation.