Algebra: domains, systems, inequalities and functions

GRE General Test

Scope and task

Original classroom practice within the reviewed shorter-test task types. This does not simulate an adaptive test or predict a scaled or writing score.

- Solve equations while preserving denominator and radical restrictions
- Analyse inequalities and simultaneous conditions without losing endpoints
- Evaluate functions and translate verbal relationships into algebra

domain restriction : A condition identifying inputs on which the original expression is defined.

simultaneous solution : Values satisfying every relation in a system at the same time.

Read the evidence and choose a method

Write restrictions before manipulating an equation. A denominator must be nonzero; a real even root needs a nonnegative argument. Cancelling factors preserves the expression only on its original domain. Squaring can introduce candidates because a square loses sign information; verify every candidate in the unsquared relation.

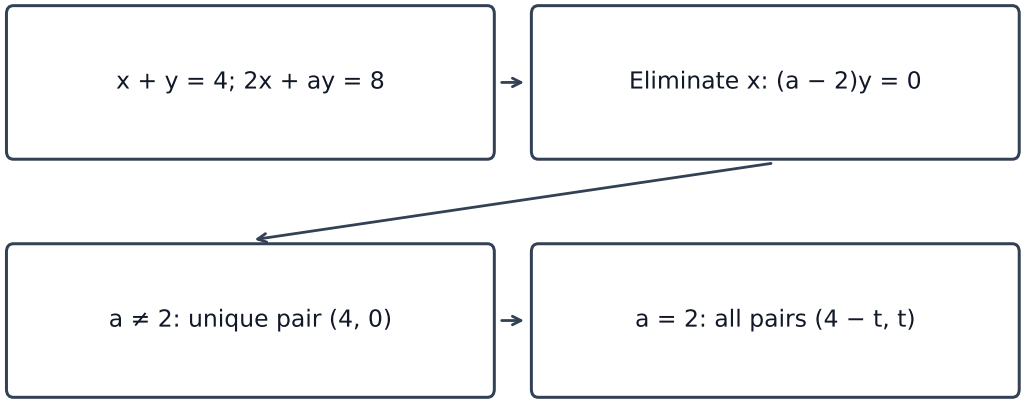
Solving a system requires the same ordered pair to satisfy all relations. Substitution or elimination can expose one, no or infinitely many linear solutions. For an inequality, multiplication or division by a negative reverses direction. For a factored quadratic inequality, order roots and test interval signs, including endpoints only when equality is allowed.

A function assigns one output to each permitted input. Evaluate the inner function first in a composition, preserving domain restrictions. Read a slope as change in output per input unit and an intercept as the output at input zero only when that input is meaningful. A verbal model should name variables and keep quantities in compatible units.

For Quantitative Comparison, preserve all allowed values. A variable with $x^2=4$ could be -2 or 2 unless constrained. One counterexample can reject a universal relationship; several favourable substitutions cannot prove it for every real value. Algebraic structure can establish a relationship without exhaustive numerical sampling.

Worked reasoning

For $\sqrt{(x+2)}=x$, $x\geq 0$. Squaring gives $x^2-x-2=0$, candidates 2 and -1. Only 2 satisfies the original. For $(x-1)(x-4)\leq 0$, an upward quadratic is nonpositive on $1\leq x\leq 4$, including roots. In the system $x+y=10$, $x-y=2$, adding gives $2x=12$; $x=6$ and $y=4$. For $f(t)=2t+1$ and $g(x)=x^2$, $f(g(3))=2\cdot 9+1=19$, applying g first.



The value of a decides between one solution and infinitely many

Conditions and common errors

A transformed equation supplies candidates, not automatic original solutions. Negative inequality multipliers and strict endpoints need explicit checks.

Original application

Solve $(x^2-4)/(x-2)=5$ on its original real domain. Solve $(x+1)(x-3)\leq 0$ and find $f(g(2))$ for $f(t)=t^2$, $g(x)=x-3$.

Model and reasoning

The rational equation excludes $x=2$. For allowed x it reduces to $x+2=5$, giving $x=3$, which substitutes correctly. The quadratic is nonpositive between its roots, including them: $-1\leq x\leq 3$. The composition applies g first: $g(2)=-1$ and $f(g(2))=1$. Cancelling a factor does not restore excluded inputs.

Independent transfer

For real a, classify the system $x + y = 4$, $2x + ay = 8$ as having one, none or infinitely many solutions. Compare with the second equation $2x + ay = 9$. Give solutions in every consistent case.

■ Check after attempting

Subtract twice the first equation. In the first system $(a - 2)y = 0$: for a not equal to two, y=0,x=4, uniquely. For a=2, both equations are the same relation, giving every pair $(4 - t, t)$. There is no inconsistent case in this first system. With right side nine, $(a - 2)y = 1$. For a not equal to two, $y = 1/(a - 2)$ and $x = 4 - 1/(a - 2)$, uniquely. For a=2 it demands 0=1, so no solution. A zero coefficient row means inconsistency only when the right side is nonzero.