Arithmetic: divisibility, remainders and multiplicative change

GRE General Test

Scope and task

Original classroom practice within the reviewed shorter-test task types. This does not simulate an adaptive test or predict a scaled or writing score.

- Use prime factorisation for divisibility, greatest common factors and least common multiples
- Interpret remainders with the original divisor
- Combine ratios and percentage changes with the correct reference total

least common multiple : The smallest positive integer divisible by each given positive integer.

remainder : The nonnegative amount left after division by a positive integer, smaller than that divisor.

Read the evidence and choose a method

An integer divisible by d can be written dk; a remainder r gives dk+r with $0 \leq r < d$ for a positive divisor. Prime factorisation identifies divisibility and common multiples. The greatest common factor takes shared prime powers at their smaller exponents; the least common multiple takes every needed prime power at its larger exponent.

Keep odd/even and positive/negative properties separate. A product of two odd integers is odd; a product with an even integer is even. A statement about integers need not hold for real numbers. For quantitative comparison, exploit a remainder form to expose possible cases rather than assume the smallest positive example is the only value.

A ratio a:b allocates a whole into a+b equal parts when the groups exhaust it. A percentage change multiplies the original value by 1+r or 1−r with r expressed as a decimal. Successive changes multiply factors. A part-of-a-part calculation changes the reference group at each stage; label the denominator before multiplying.

Estimate magnitude and check integer restrictions after solving. Fractions, roots and negative exponents follow their ordinary domain restrictions; even roots represent the nonnegative principal value. Do not cancel a term in a sum as if it were a factor. Exact arithmetic can be faster and safer than decimal calculator entries.

Worked reasoning

Known: $18 = 2 \cdot 3^2$ and $24 = 2^3 \cdot 3$. Shared smaller powers give GCF six; needed larger powers give LCM seventy-two. If $n = 5k + 2$, then $2n = 5(2k) + 4$, so the remainder on division by five is four.

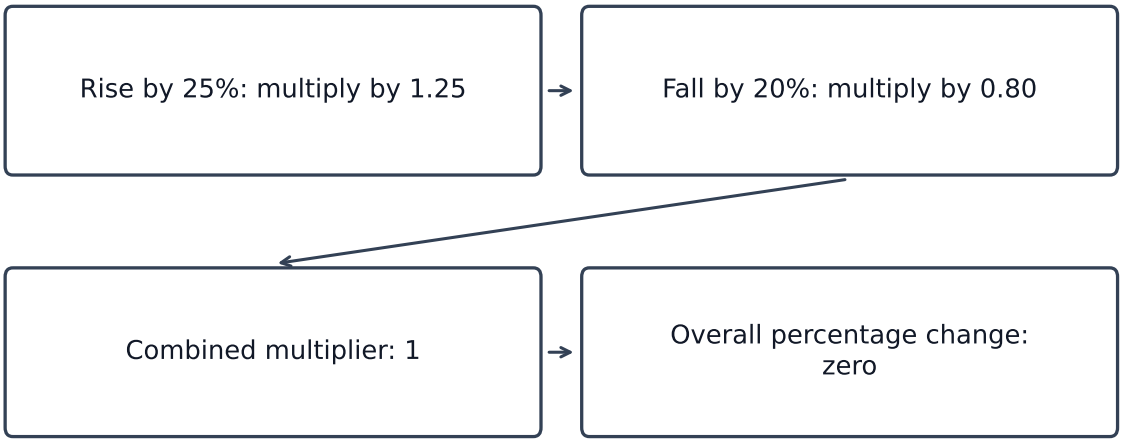
For a model price of 100 CNY decreased then increased by 20%, multiply the two changes:

$$P_{\text{final}} = P_{\text{initial}}(1 - 0.20)(1 + 0.20).$$

$$P_{\text{final}} = (100 \text{ CNY})(0.8)(1.2) = 96 \text{ CNY}.$$

The price is not restored because the second percentage uses a different base.

Dividing 250 in the ratio 2:3 gives 100 and 150: each of five parts is fifty.



×1.25 then ×0.80 gives ×1, so the overall change is zero

Conditions and common errors

Equal percentage increases and decreases use different bases. A remainder must be reported relative to the named divisor.

Original application

Find the greatest common factor and least common multiple of 36 and 60. A price rises 25% then falls 20%. Find its overall percentage change.

■ Model and reasoning

$36 = 2^2 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$. Shared smaller powers give GCF=12; larger required powers give LCM=180. Their product $12 \cdot 180 = 36 \cdot 60$ checks the result. The price multiplier is $1.25 \cdot 0.80 = 1$, so the overall change is zero percent. The two changes use different reference prices; adding +25 and -20 would give a wrong five-percent increase.

Independent transfer

An integer n has remainder 5 on division by 8. Find the remainder of $3n+7$ on division by 8, including for negative n. A value rises 10% in one year and 20% the next. What single percentage reduction restores its original value?

■ Check after attempting

Write $n = 8k + 5$ for integer k. Then $3n + 7 = 24k + 22 = 8(3k + 2) + 6$, so the remainder is six for all integer k. The final value is 1.32 times the original. Restoring it multiplies by $1/1.32 = 25/33$, a reduction of $1 - 25/33 = 8/33$, or about 24.24%. A 30% reduction does not reverse compounded growth. The remainder convention uses $0 \le r < 8$ even when n is negative.