

# 11.1 Arithmetic: divisibility, remainders and multiplicative change

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Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

**KEY WORDS** least common multiple • remainder

## Objective

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Use prime factorisation for divisibility, greatest common factors and least common multiples; Interpret remainders with the original divisor; Combine ratios and percentage changes with the correct reference total.

Original public tasks and locally authored marking guidance. No official score is assigned.

## 1 Key knowledge and worked examples

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**least common multiple** : The smallest positive integer divisible by each given positive integer.

**remainder** : The nonnegative amount left after division by a positive integer, smaller than that divisor.

An integer divisible by  $d$  can be written  $dk$ ; a remainder  $r$  gives  $dk+r$  with  $0 \leq r < d$  for a positive divisor. Prime factorisation identifies divisibility and common multiples. The greatest common factor takes shared prime powers at their smaller exponents; the least common multiple takes every needed prime power at its larger exponent.

Keep odd/even and positive/negative properties separate. A product of two odd integers is odd; a product with an even integer is even. A statement about integers need not hold for real numbers. For quantitative comparison, exploit a remainder form to expose possible cases rather than assume the smallest positive example is the only value.

A ratio  $a:b$  allocates a whole into  $a+b$  equal parts when the groups exhaust it. A percentage change multiplies the original value by  $1+r$  or  $1-r$  with  $r$  expressed as a decimal. Successive changes multiply factors. A part-of-a-part calculation changes the reference group at each stage; label the denominator before multiplying.

Estimate magnitude and check integer restrictions after solving. Fractions, roots and negative exponents follow their ordinary domain restrictions; even roots represent the nonnegative principal value. Do not cancel a term in a sum as if it were a factor. Exact arithmetic can be faster and safer than decimal calculator entries.

Known:  $18 = 2 \cdot 3^2$  and  $24 = 2^3 \cdot 3$ . Shared smaller powers give GCF six; needed larger powers give LCM seventy-two. If  $n = 5k + 2$ , then  $2n = 5(2k) + 4$ , so the remainder on division by five is four.

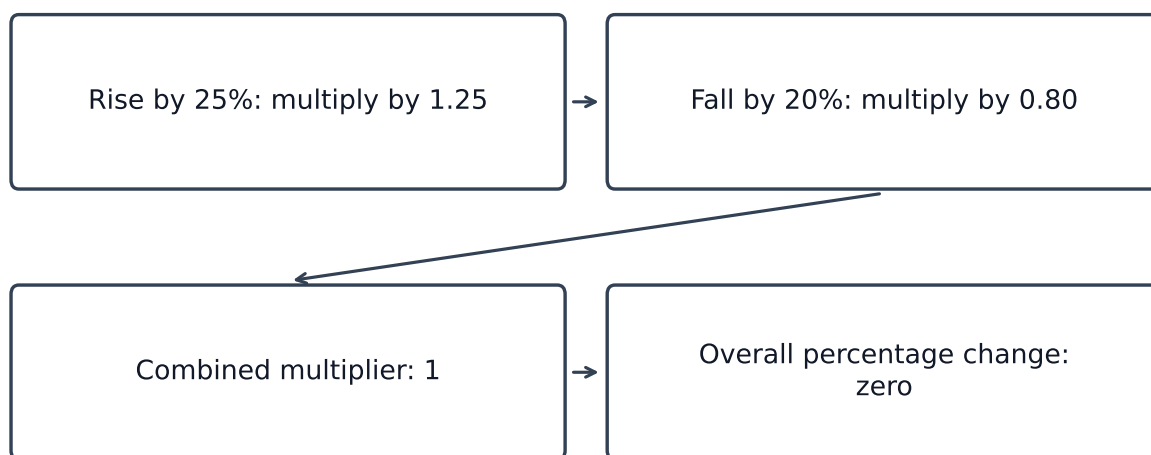
For a model price of 100 CNY decreased then increased by 20%, multiply the two changes:

$$P_{\text{final}} = P_{\text{initial}}(1 - 0.20)(1 + 0.20).$$

$$P_{\text{final}} = (100 \text{ CNY})(0.8)(1.2) = 96 \text{ CNY}.$$

The price is not restored because the second percentage uses a different base.

Dividing 250 in the ratio 2:3 gives 100 and 150: each of five parts is fifty.



*$\times 1.25$  then  $\times 0.80$  gives  $\times 1$ , so the overall change is zero*

## 2 Practice

**2.1** Find the greatest common factor and least common multiple of 36 and 60. A price rises 25% then falls 20%. Find its overall percentage change.

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### 3 Exam-style questions

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**3.1** An integer  $n$  has remainder 5 on division by 8. Find the remainder of  $3n + 7$  on division by 8, including for negative  $n$ . A value rises 10% in one year and 20% the next. What single percentage reduction restores its original value?

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