

GAC016 Mathematics III: Calculus

GAC016-aligned calculus practice; the centre's brief sets the actual assessment and grading.

Precision about change and accumulation

An average speed does not tell you every instantaneous speed, and a signed displacement does not always equal total distance.

Notation and interpretation

- State the independent variable, domain and units.
- dy/dx is a derivative with respect to x ; a dot over y normally denotes a derivative with respect to time.
- They describe the same operation only when their independent variables agree.
- A negative rate of change of 3 cubic centimetres per second means the quantity is falling, not that it is negative.

The derivative, and its definition

- A derivative is a rate of change: how fast y changes as x changes.
- Geometrically it is the gradient of the tangent at a point.
- First principles uses the difference quotient for nonzero h , then takes its limit.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

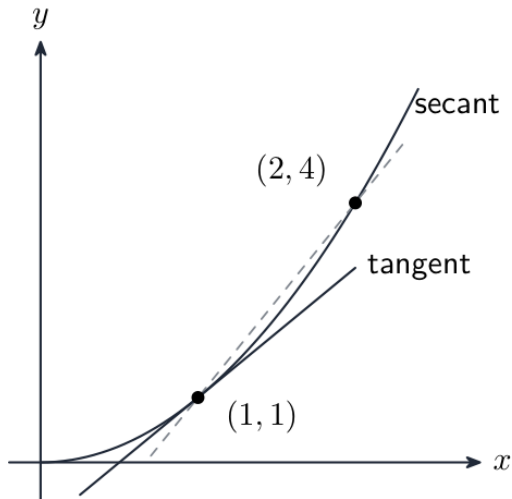
The differentiation rulebook

Rule	Statement
Power rule	$y = x^n$ gives $dy/dx = n x^{(n-1)}$, on a domain where the power is differentiable
Product rule	$(uv)' = u'v + uv'$
Quotient rule	$(u/v)' = (u'v - uv')/v^2$, where v is nonzero
Chain rule	$y = f(g(x))$ gives $dy/dx = f'(g(x))$ times $g'(x)$

Stationary points and classification

- Stationary points occur where $f'(x) = 0$.
- The second derivative classifies a stationary point when it is nonzero.
- Negative gives a local maximum; positive gives a local minimum.
- A zero second derivative is inconclusive; use derivative signs or other evidence.

Worked example: secant versus tangent



- Known: $f(x) = x^2$; the secant joins $(1, 1)$ to $(2, 4)$.
- Secant slope: $(4-1)/(2-1) = 3$.
- First principles: $f'(x) = 2x$, so the tangent slope at $x = 1$ is 2.

Your turn: another point on the same curve

Your turn

For $f(x) = x$ squared, what is the tangent slope at $x = 3$?

- A. 6, from $f'(x) = 2x$
- B. 9, the value of the function
- C. 5, the average slope from 1 to 3
- D. 3, the value of x

Indefinite, definite, and the constant

Kind	What it gives
Indefinite integral	An antiderivative family, plus the constant c : $\int x^n dx = x^{n+1}/(n+1) + c$, for $n \neq -1$
Definite integral	A number: $\int_a^b f(x) dx = F(b) - F(a)$
Substitution	Reverses the chain rule for a composed integrand

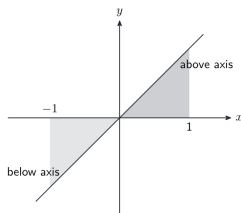
Use the evidence

For a continuous integrand, the fundamental theorem of calculus connects the definite integral to antiderivative values at its limits.

The plus c warning

Forgetting $+c$ on an indefinite integral is the module's most frequent lost mark — on questions otherwise answered correctly.

Worked example: cancellation of signed areas



- Known: $f(x) = x$, from -1 to 1 .
- Signed integral: 0 (areas cancel). Geometric area: $1/2 + 1/2 = 1$.

$$I = \int_{-1}^1 x \, dx = 0; \quad A = \frac{1}{2} + \frac{1}{2} = 1$$

Your turn: a one-sided area

Your turn

What is the signed integral of $y = x$ from 0 to 3?

- A. $9/2$, from x squared over 2 at 3 minus at 0
- B. 0, by cancellation
- C. 9, the value of the function at 3
- D. 3, the interval length

Optimisation is a method, not a guess

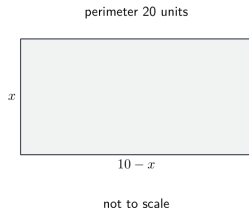
- Write the quantity as a function of one variable on a stated feasible domain.
- Differentiate, find the candidates, and justify the required optimum.
- Check included boundaries and integer constraints where relevant.

Rates chain together

- Rates of change chain: $dV/dt = (dV/dr)$ times (dr/dt) .
- Area between two curves is the integral of $(f - g)$, with f the upper curve on the whole interval.
- Split at crossings if their order changes.
- Marginal cost and revenue are derivatives of stated continuous models — instantaneous rates, not exact discrete-unit changes.

$$dV/dt = (dV/dr) \cdot (dr/dt)$$

Worked example: the fixed-perimeter rectangle



- Perimeter 20: width x , length $10 - x$, domain $0 < x < 10$.
- $A'(x) = 10 - 2x$ gives $x = 5$; $A'' < 0$ and zero-area ends make it the global maximum.

$$A(x) = x(10 - x); \quad A'(x) = 10 - 2x; \quad x = 5, \quad A = 25$$

Your turn: justify the optimum

Your turn

In the perimeter-20 rectangle, why is $x = 5$ the global maximum, not just a stationary point?

- A. Because $A'(5) = -2 < 0$, and the area approaches zero at both domain ends
- B. Because stationary points are always maxima
- C. Because 5 is the middle of the domain
- D. Because squares are always optimal

Check yourself

- What is the derivative of x squared, and what is the secant slope from $(1,1)$ to $(2,4)$?
- Why does the integral of $y = x$ over -1 to 1 give 0 , and what is the geometric area?
- Which single omission loses the most marks on indefinite integrals?
- Why does a fixed perimeter leave only one area variable?