

4.5 Analyse simple graphs, trees and weighted routes

Name: _____ Class: _____ Date: _____ **Total: 24 marks**

KEY WORDS vertices 顶点 • edges 边 • tree 树

Objective

Analyse simple graphs, trees and weighted routes. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

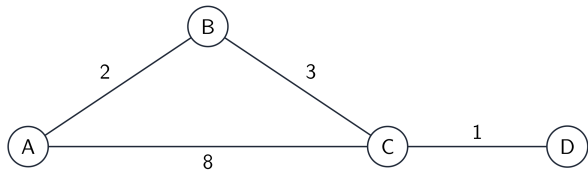
1 Key knowledge and worked examples

■ **Key knowledge: state whether connections are directed, weighted or undirected**

A graph has **vertices** 顶点 connected by **edges** 边. This sheet uses finite simple undirected graphs unless a question says otherwise: no loops or repeated edges. Degree counts incident edges; the sum of degrees is twice the edge count. A path connects vertices, and a cycle returns to its start without repeating other vertices.

A **tree** 树 is connected and has no cycles; a tree with n vertices has n minus 1 edges. That edge count alone does not prove a graph is a tree without connectedness. In a weighted network, a shortest path minimises total path weight, whereas a minimum spanning tree connects all vertices with minimum total edge weight and no cycle. These are different optimisation tasks.

■ **Worked example: compare total route weight, not the number of edges**



Known edge weights are $AB = 2$, $BC = 3$, $AC = 8$ and $CD = 1$. The path A-C-D has weight 9, while A-B-C-D has weight 6. Therefore the three-edge path is shorter by weight despite having more edges. The minimum spanning tree for this small network uses AB, BC and CD with total 6; the agreement of totals here does not make the tasks identical.

2 Practice

2.1 A simple undirected graph has edges AB, AC and BC. Find the degree of A and the total edge count. [2]

2.2 A tree has 9 vertices. Find its edge count and state the property excluding a triangle cycle. [2]

2.3 A simple undirected graph has degrees 3, 2, 2, 1. Find its edge count and explain why degree sum 9 would be impossible. [3]

2.4 A graph has edges AB, BC, CA and DE, on exactly five vertices A to E. It has n minus 1 edges. Decide whether it is a tree and justify your decision using two graph properties. [3]

3 Exam-style questions

3.1 A weighted undirected network has edges $AB = 2$, $AC = 6$, $BC = 1$, $BD = 7$, $CD = 3$. (a) Find a shortest path from A to D and its weight. (b) Find a minimum spanning tree and its total weight. (c) Explain the different requirements of these tasks. [6]

3.2 A weighted undirected network on A, B, C, D and E has edges $AB = 1$, $AC = 4$, $BC = 2$, $BD = 5$, $CD = 1$, $CE = 6$, $DE = 3$. (a) Find a minimum spanning tree and its total weight, showing how cycles are avoided. (b) Find a shortest path from A to E and its weight. (c) If DE closes, find a new shortest A-to-E route using the remaining edges. Explain why that new route must be checked rather than retaining the old total. [8]