

Solutions

2.1

A is incident to AB and AC, so degree 2. Total edge count 3.

2.2

A tree has $n - 1 = 9 - 1 = 8$ edges. It is acyclic, so no triangle cycle is allowed.

2.3

Each edge contributes two to total degree.

$$|E| = \frac{\sum d(v)}{2} = \frac{3 + 2 + 2 + 1}{2} = 4$$

Four edges. A sum of 9 is odd and cannot equal twice an integer edge count.

2.4

It is not a tree. The ABC triangle is a cycle. The DE component is disconnected from ABC. The four-edge count alone is insufficient.

3.1

(a) A-B-C-D has weight $2 + 1 + 3 = 6$. Correct path earns B1 and weight B1. The other simple A-to-D alternatives are A-B-D with 9, A-C-D with 9, and A-C-B-D with 14, so 6 is shortest.

(b) Choose BC, AB and CD to connect all vertices without a cycle. Total weight 6.

(c) The shortest-path task connects only the specified endpoints with minimum route weight. The spanning-tree task connects all vertices with no cycles and minimum total included edge weight.

3.2

(a) Select AB and CD, both weight 1. Add BC with weight 2, joining those components without a cycle. Add DE with weight 3 to connect E. Total $W = 1 + 1 + 2 + 3 = 7$, with all five vertices connected and four edges.

(b) Shortest path A-B-C-D-E. Weight $W_p = 1 + 2 + 1 + 3 = 7$. Nonnegative edge weights and comparison with alternative approaches to C/D establish no shorter route.

(c) E is now accessible only through CE, so the shortest remaining route is A-B-C-E with weight $1 + 2 + 6 = 9$. Correct route and weight. The closed edge cannot be used; the old path is infeasible and its total is not a valid remaining-network answer.