

Solutions

2.1

Intersection is $\{3\}$. Union is $\{1, 2, 3, 4\}$. Do not repeat 3 in a set.

2.2

The complement is $U \setminus A = \{2, 4, 6\}$. Seven is outside the universe, so it is not in this complement.

2.3

Range is $\{a, b\}$. It is a function. Every domain input has exactly one output, although two inputs share a.

2.4

Use inclusion-exclusion.

$$|A \cup B| = |A| + |B| - |A \cap B| = 15 + 12 - 5 = 22$$

Union count 22.

$$N_{outside} = |U| - |A \cup B| = 30 - 22 = 8$$

Outside count 8.

3.1

Music only is $23 - 7 = 16$. Drama only is $18 - 7 = 11$. Inclusion-exclusion gives the union.

$$|M \cup D| = 23 + 18 - 7 = 34$$

Neither is $40 - 34 = 6$. Exactly one is $16 + 11 = 27$. The disjoint regions sum to $16 + 7 + 11 + 6 = 40$.

3.2

(a) Range is $\{0, 1, 4\}$.

(b) f is a function because each input has exactly one square. It is not one-to-one. Distinct inputs such as negative 2 and 2 both give 4.

(c) The inverse relation is not a function on the original range because 1 and 4 each correspond to two domain values.

(d) The restricted function is one-to-one and has the same range $\{0, 1, 4\}$. Its inverse is the nonnegative square root on that range. Checks: 0 maps back to 0, 1 to 1, and 4 to 2. The restriction is essential; do not silently discard the original negative inputs.