

3.3 Apply calculus with physical and feasible-domain checks

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS optimisation 最优化 • rates of change 变化率 • marginal cost 边际成本

Objective

Apply calculus with physical and feasible-domain checks. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

1 Key knowledge and worked examples

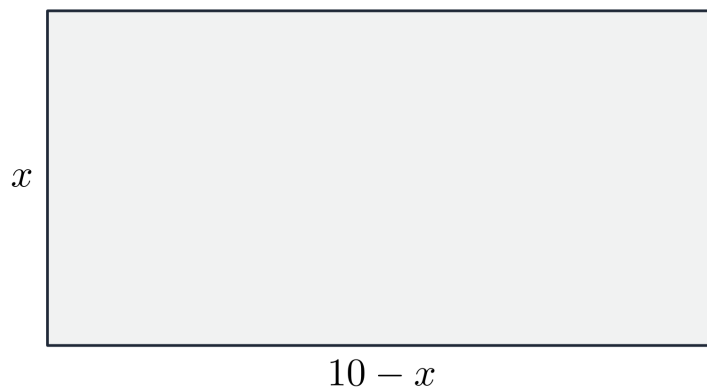
■ **Key knowledge:** a stationary candidate is not automatically the required optimum

Optimisation 最优化 starts with a quantity and its feasible domain. Use a constraint to express it in one variable, find stationary candidates, compare boundaries where applicable, and justify the required maximum or minimum. A continuous optimum may differ from the best feasible integer choice.

Related **rates of change** 变化率 use the chain rule: $dV/dt = (dV/dr)(dr/dt)$ when V depends on r and r on time. **Marginal cost** 边际成本 is the derivative of a continuous cost model, an instantaneous rate rather than automatically an exact discrete-unit increase. Integrating a rate gives an accumulated change; include an initial value when finding a final amount. All scenarios here are invented mathematical models.

■ Worked example: a fixed perimeter leaves one area variable

perimeter 20 units



not to scale

Known: a rectangle has perimeter 20, so length plus width is 10. Let width be x ; length is $10 - x$ and $0 < x < 10$.

$$A(x) = x(10 - x)$$

$$A'(x) = 10 - 2x$$

The stationary candidate is $x = 5$. Since $A''(x) = -2 < 0$ and the area approaches zero at both domain ends, this candidate is the global maximum. It is a 5 by 5 square with area 25.

2 Practice

2.1 A continuous model has cost $C(q) = 100 + 3q + 0.2q^2$. Find its marginal cost at q equal to 10. [2]

2.2 A constant flow adds water at $r(t) = 6$ litres per minute for 5 minutes. Find the increase in volume. [2]

2.3 A circle radius grows at $dr/dt = 0.5$ cm per second. Find its area growth rate when r equals 4 cm. [3]

2.4 For $C(q) = 100 + 3q + 0.2q^2$, compare $C(11) - C(10)$ with $C'(10)$. Explain the difference. [3]

3 Exam-style questions

3.1 A rectangle has perimeter 40 m. (a) Express area in terms of width x , state its feasible domain, and find the width maximising area. (b) Justify the maximum and find the maximum area. [6]
