

Solutions

2.1

Differentiate the cost model.

$$C'(q) = 3 + 0.4q$$

$$C'(10) = 3 + 0.4(10) = 7$$

Marginal cost 7 cost units per quantity unit.

2.2

Integrate the rate over the stated time.

$$\Delta V = \int_0^5 r(t) dt = \int_0^5 6 dt = 30 \text{ litres}$$

Increase 30 litres.

2.3

Use $A = \pi r^2$ with the chain rule.

$$\frac{dA}{dt} = \frac{dA}{dr} \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$$

Correct related-rate relation.

$$\frac{dA}{dt} = 2\pi(4)(0.5) = 4\pi \frac{\text{cm}^2}{\text{s}}$$

Correct rate.

2.4

Compute the actual one-unit change.

$$\Delta C = C(11) - C(10) = 157.2 - 150 = 7.2$$

Correct finite increase 7.2. The derivative at 10 is 7, an instantaneous rate; the derivative changes over the next unit, so it need not equal the finite increase.

3.1

The length is $20 - x$, with $0 < x < 20$.

$$A(x) = x(20 - x)$$

Correct area model. Differentiate and set to zero.

$$A'(x) = 20 - 2x = 0 \implies x = 10$$

Width 10 m. The second derivative is negative 2 and area approaches zero at the endpoints, so this is the global maximum on the feasible domain.

$$A_{max} = 10(20 - 10) = 100 \text{ m}^2$$

Maximum area.

3.2

(a) Differentiate.

$$P'(q) = -2q + 11 = 0 \implies q = 5.5$$

Stationary quantity. Since $P''(q) = -2 < 0$ and both boundary profits are smaller, it is the global maximum.

$$P(5.5) = -(5.5)^2 + 11(5.5) - 10 = 20.25$$

Continuous maximum profit.

(b) Check the adjacent integers 5 and 6.

$$P(5) = -25 + 55 - 10 = 20$$

$$P(6) = -36 + 66 - 10 = 20$$

Both quantities maximise feasible integer profit.

(c) Factor the zero-profit equation: $P(q) = -(q - 1)(q - 10) = 0$, giving break-even quantities 1 and 10. Strictly positive profit is $1 < q < 10$, hence whole numbers 2 through 9.