

3.2 Find integrals and distinguish signed accumulation from area

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS indefinite integral 不定积分 • definite integral 定积分
• constant of integration 积分常数

Objective

Find integrals and distinguish signed accumulation from area. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

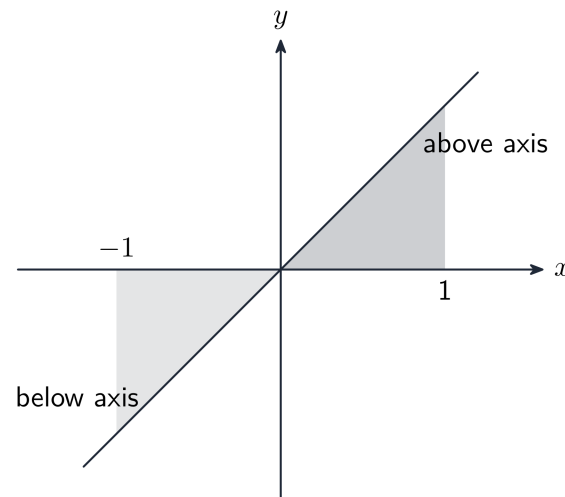
1 Key knowledge and worked examples

■ **Key knowledge: a definite integral can cancel contributions with opposite signs**

An **indefinite integral** 不定积分 gives a family of antiderivatives with a **constant of integration** 积分常数 C . The power rule is $\int x^n dx = x^{n+1}/(n+1) + C$ for $n \neq -1$, on a suitable domain. For the reciprocal, $\int 1/x dx = \ln|x| + C$ on an interval not crossing zero.

A **definite integral** 定积分 is signed accumulation. When an antiderivative F is valid over the interval, $\int_a^b f(x) dx = F(b) - F(a)$. Geometric area is nonnegative, so split where a curve changes sign. Substitution reverses the chain rule and requires matching differential factors; definite-integral bounds must change consistently or return to the original variable.

■ **Worked example: negative and positive areas can cancel**



Known: $f(x) = x$ from negative 1 to 1. Its definite integral is zero because the negative and positive contributions cancel.

$$I = \int_{-1}^1 x dx = \left[\frac{x^2}{2} \right]_{-1}^1 = \frac{1}{2} - \frac{1}{2} = 0$$

Geometric area instead adds the magnitudes of the two triangular regions.

$$A = -\int_{-1}^0 x dx + \int_0^1 x dx = \frac{1}{2} + \frac{1}{2} = 1$$

2 Practice

2.1 Find $\int (3x^2 - 4) dx$.

[2]

2.2 Evaluate $\int_0^2 (2x + 1) \, dx$. [2]

2.3 Find $\int 6x/(3x^2 + 2) \, dx$ using substitution. [3]

2.4 Find both the signed integral $\int_{-1}^1 x \, dx$ and the geometric area between that line and the x-axis over the same interval. [3]

3 Exam-style questions

3.1 A particle has velocity $v(t) = 4 - t$ metres per second for $0 \leq t \leq 6$ seconds and starts at position 5 m. Find (a) its displacement, (b) its total distance travelled, and (c) its final position. [6]

[8]