

Solutions

2.1

Use the power rule term by term.

$$F(x) = x^3 - 4x + C$$

Correct antiderivative including C.

2.2

Use antiderivative $F(x) = x^2 + x$ and subtract the lower-bound value.

$$I = [x^2 + x]_0^2 = (2^2 + 2) - (0^2 + 0) = 6$$

Value 6.

2.3

Let $u = 3x^2 + 2$, so $du = 6x \, dx$.

$$I = \int \frac{1}{u} du = \ln |u| + C$$

Logarithmic antiderivative. Since $3x^2 + 2 > 0$ for real x , $I = \ln(3x^2 + 2) + C$.

2.4

The signed integral is $[x^2/2]_{-1}^1 = 0$. Split the geometric area at zero and reverse the sign of the lower region.

$$A = - \int_{-1}^0 x \, dx + \int_0^1 x \, dx = 1$$

Area 1.

3.1

(a) Integrate velocity for signed displacement.

$$\Delta s = \int_0^6 v(t) dt = \left[4t - \frac{t^2}{2} \right]_0^6 = 6 \text{ m}$$

Displacement 6 m.

(b) Velocity changes sign at $t = 4$ s. Split there and add magnitudes.

$$D = \int_0^4 (4 - t) dt - \int_4^6 (4 - t) dt = 8 + 2 = 10 \text{ m}$$

Distance 10 m.

(c) Final position $s_f = s_0 + \Delta s = 5 + 6 = 11$ m.

3.2

(a) Equate the curve values.

$$x = x^2 - 2x \implies x(x - 3) = 0$$

Intersections at x equal to 0 and 3.

(b) The difference is $x - (x^2 - 2x) = x(3 - x) > 0$ for $0 < x < 3$, so the line is upper.

(c) Integrate upper minus lower.

$$A = \int_0^3 [x - (x^2 - 2x)] dx = \int_0^3 (3x - x^2) dx$$

$$A = \left[\frac{3}{2}x^2 - \frac{1}{3}x^3 \right]_0^3 = \frac{9}{2}$$

Correct antiderivative. Area $9/2$ square units.

(d) Each half has area $\int_0^{3/2} (3x - x^2) dx = 9/4$ and $\int_{3/2}^3 (3x - x^2) dx = 9/4$. Correct half integrals. Their sum is $9/2$, matching the result.