

3.1 Differentiate functions and classify local behaviour

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS derivative 导数 • chain rule 链式法则 • stationary point 驻点

Objective

Differentiate functions and classify local behaviour. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

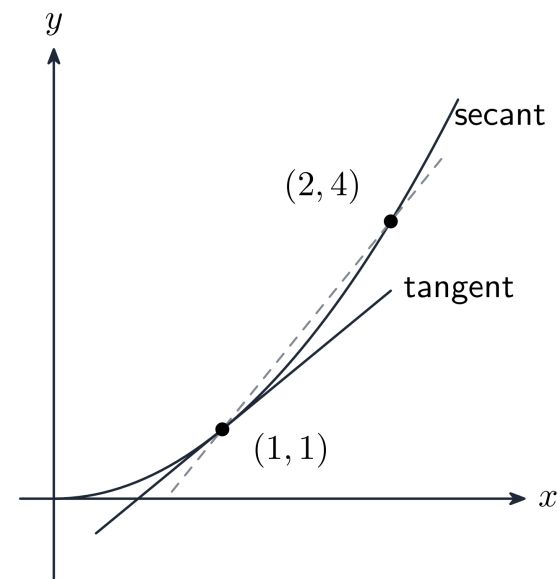
1 Key knowledge and worked examples

■ Key knowledge: a derivative exists only when the limiting slope exists

A **derivative** 导数 is a limiting rate of change, $f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)]/h$, when that limit exists. For a differentiable curve it is the tangent gradient. A sharp corner can prevent differentiability. A dot over a variable normally denotes a time derivative, not an arbitrary derivative with respect to x .

Use the power, product, quotient and **chain rule** 链式法则 for the function's form and domain. The quotient rule needs a nonzero denominator; trigonometric derivative rules here use radians. A **stationary point** 驻点 has derivative zero. A positive second derivative at it gives a local minimum and a negative one a local maximum; a zero second derivative leaves that test inconclusive. Check derivative signs or other evidence.

■ Worked example: a tangent slope differs from a secant slope



Known: $f(x) = x^2$. The secant between x equal to 1 and 2 has slope 3, but the derivative at x equal to 1 is 2.

$$m_{sec} = \frac{f(b) - f(a)}{b - a} = \frac{4 - 1}{2 - 1} = 3$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} (2x + h) = 2x$$

$$m_{tan} = f'(1) = 2(1) = 2$$

The average slope over a finite interval need not equal the instantaneous slope at either endpoint.

2 Practice

2.1 Differentiate $y = 4x^3 - 6x^2 + 5$. [2]

2.2 Differentiate $y = (2x + 1)^3$. [2]

2.3 Differentiate $y = x^2 \sin x$, where x is in radians. [3]

2.4 For $y = (2x + 1)/(x - 1)$, state the excluded x -value and find the derivative. [3]

3 Exam-style questions

3.1 Use difference quotients to examine $f(x) = |x|$ at x equal to 0. (a) Find the right-hand and left-hand limiting slopes. (b) Decide whether a derivative exists at 0. (c) Give the derivative for x greater than 0 and x less than 0. [6]

[8]