

Solutions

2.1

Apply the power rule term by term; a constant has derivative zero.

$$\frac{dy}{dx} = 12x^2 - 12x$$

Correct derivative.

2.2

Use the chain rule, including inner derivative 2.

$$\frac{dy}{dx} = 3(2x + 1)^2(2) = 6(2x + 1)^2$$

Correct derivative.

2.3

Use the product rule.

$$y' = u'v + uv'$$

$$y' = 2x \sin x + x^2 \cos x$$

Correct first product contribution. Correct second contribution and their sum.

2.4

Exclude $x = 1$ because the denominator is zero. Use the quotient rule.

$$y' = \frac{u'v - uv'}{v^2}$$

$$y' = \frac{2(x - 1) - (2x + 1)}{(x - 1)^2} = \frac{-3}{(x - 1)^2}$$

Correct derivative on the stated domain.

3.1

At zero, the difference quotient is $|h|/h$. For $h > 0$ it equals 1, so the right-hand limit is 1. For $h < 0$ it equals negative 1, so the left-hand limit is negative 1. The two one-sided limits differ, so no derivative exists at zero. For positive x , $f'(x) = 1$. For negative x , $f'(x) = -1$. The function itself is still continuous at zero.

3.2

Differentiate.

$$f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$$

Correct factorisation. Stationary x-values are 1 and 3.

$$f(1) = 1 - 6 + 9 + 1 = 5$$

$$f(3) = 27 - 54 + 27 + 1 = 1$$

Points (1, 5) and (3, 1). Use the second derivative to classify them.

$$f''(x) = 6x - 12$$

At 1 it is negative 6, giving a local maximum; at 3 it is 6, giving a local minimum. Both classifications correct. The first derivative is positive for $x < 1$ and $x > 3$, so increasing on those intervals. It is negative for $1 < x < 3$, so decreasing there.