

Solutions

2.1

Standardise using the model mean and standard deviation.

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2$$

Z-score 2.

2.2

Use the complement of the area at or below 1.

$$P(Z > 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587$$

Probability 0.1587.

2.3

Symmetry gives $\Phi(-1) = 1 - 0.8413 = 0.1587$. Subtract cumulative areas.

$$P(-1 < Z < 1) = \Phi(1) - \Phi(-1) = 0.8413 - 0.1587 = 0.6826$$

Probability 0.6826.

2.4

Invert standardisation: $x = \mu + z\sigma$.

$$x_1 = 70 + (-1)(8) = 62$$

Lower raw value.

$$x_2 = 70 + 1.5(8) = 82$$

Upper raw value.

$$P(X > 82) = 1 - \Phi(1.5) = 1 - 0.9332 = 0.0668$$

Tail probability.

3.1

(a) Standardise both bounds.

$$z_L = \frac{x_L - \mu}{\sigma} = \frac{35 - 40}{5} = -1$$

$$z_U = \frac{x_U - \mu}{\sigma} = \frac{50 - 40}{5} = 2$$

Both bounds correct. Subtract cumulative areas.

$$p = \Phi(2) - \Phi(-1) = 0.9772 - 0.1587 = 0.8185$$

Interval probability.

(b) Tail probability is $1 - 0.9772 = 0.0228$, so expected count $E = Np = 200(0.0228) = 4.56$. The random observed count is an integer and can vary; 4.56 is an expectation over repeated modelled samples.

3.2

(a) Standardise relative to each model.

$$z_A = \frac{80 - 60}{10} = 2$$

A z-score.

$$z_B = \frac{85 - 75}{5} = 2$$

B z-score. Both scores are two model standard deviations above their respective means.

(b) For each model, $p = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$.

(c) Expected count on each scale is $E = Np = 500(0.0228) = 11.4$.

(d) Normality and representative parameters must be justified for the actual data. Equal standardised relative position does not establish equivalent test content, difficulty, reliability or learner ability; separate evidence is needed.