

Solutions

2.1

Favourable outcomes are 5 and 6 among six equally likely outcomes.

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Probability one third.

2.2

Complement probabilities sum to 1.

$$P(A^c) = 1 - P(A) = 1 - 0.35 = 0.65$$

Probability 0.65.

2.3

Subtract the double-counted overlap.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$$

Union probability 0.7. They are independent because $P(A)P(B) = 0.4(0.5) = 0.2 = P(A \cap B)$.

2.4

Order matters for a line.

$$N_{line} = 5! = 5(4)(3)(2)(1) = 120$$

120 orders. For an unordered pair, divide ordered choices by two:

$$N_{team} = \binom{5}{2} = \frac{5(4)}{2(1)} = 10$$

10 teams.

3.1

(a) The first red leaves two red among four counters.

$$P(RR) = P(R_1)P(R_2 | R_1) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

Both-red probability.

(b) Exactly one red has disjoint paths RB and BR.

$$P(RB) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

$$P(BR) = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$$

Correct path products.

$$P(\text{exactly one red}) = P(RB) + P(BR) = \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$$

Final probability. The remaining counts depend on which colour was removed, so the draws are not independent.

3.2

(a) Add late counts and divide by the class size.

$$P(L) = \frac{6 + 2}{40} = 0.20$$

Late probability.

(b) Restrict the denominator to bus users.

$$P(L \mid B) = \frac{6}{24} = 0.25$$

Conditional probability.

(c) Restrict the denominator to late students.

$$P(B \mid L) = \frac{6}{6 + 2} = 0.75$$

Conditional probability.

(d) Compare $P(L \mid B)$ with $P(L)$, or compare the joint probability with the product. They differ, so the events are not independent in the supplied table. This observational one-day table does not control other factors or randomly assign transport; association alone does not establish causation.