

Solutions

2.1

The constant difference is 4. Use the nth-term formula.

$$u_n = a + (n - 1)d$$

$$u_{10} = 5 + (10 - 1)(4) = 41$$

Tenth term 41.

2.2

The constant ratio is 2.

$$u_n = ar^{n-1}$$

$$u_6 = 3(2)^{6-1} = 96$$

Sixth term 96.

2.3

A sum includes every term, not only the last.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{20} = \frac{20}{2}[2(7) + (20 - 1)(3)]$$

Correct substitution.

$$S_{20} = 10(14 + 57) = 710$$

Sum 710.

2.4

Use the finite geometric sum.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_5 = \frac{16(1 - (1/2)^5)}{1 - 1/2} = 31$$

Finite sum 31. Since $|r| < 1$, $S_\infty = a/(1 - r) = 16/(1/2) = 32$.

3.1

(a) Row counts form an arithmetic sequence.

$$u_n = a + (n - 1)d$$

$$u_{15} = 12 + (15 - 1)(2) = 40$$

Row 15 contains 40 seats.

(b) Add row counts.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{15} = \frac{15}{2}[24 + 28] = 390$$

Total 390.

(c) Express the total as a function of n.

$$S_n = \frac{n}{2}[24 + 2(n - 1)] = n(n + 11)$$

Check consecutive complete-row totals.

$$S_{15} = 15(26) = 390$$

$$S_{16} = 16(27) = 432$$

The totals increase for positive n, so the smallest count is 16 rows.

3.2

(a) Ratio is $r = u_2/u_1 = 60/80 = 0.75$.

$$u_n = ar^{n-1}$$

$$u_5 = 80(0.75)^4 = 25.3125$$

Fifth term.

(b) Use the geometric sum.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_5 = \frac{80(1 - 0.75^5)}{1 - 0.75} = 244.0625$$

Five-term sum.

(c) The ratio satisfies $|r| < 1$.

$$S_\infty = \frac{a}{1 - r} = \frac{80}{1 - 0.75} = 320$$

Infinite sum 320. The remaining terms are positive, so adding them increases the sum.

(d) The second ratio is 1.2, so its positive terms grow and $|r| < 1$ fails; there is no finite infinite sum. An infinite-horizon model is not a promise about actual deliveries.