

1.6 Use exponent laws, logarithms and exponential models

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS exponent 指数 • logarithm 对数 • exponential decay 指数衰减

Objective

Use exponent laws, logarithms and exponential models. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

1 Key knowledge and worked examples

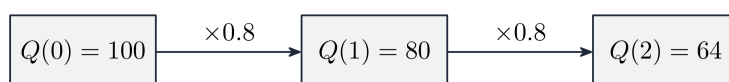
■ Key knowledge: a logarithm finds the exponent, subject to its domain

An **exponent** 指数 tells a power. For positive a , $a^m a^n = a^{m+n}$, $a^m / a^n = a^{m-n}$ and $(a^m)^n = a^{mn}$. A zero power is 1 for a nonzero base. A negative power is a reciprocal, not a negative result.

A **logarithm** 对数 answers an exponent question: $\log_b x = y$ means $b^y = x$, with $b > 0$, $b \neq 1$, and $x > 0$ for real logarithms. Product, quotient and power laws do not give a rule for the logarithm of a sum. Natural logarithms have base e .

A model $Q(t) = Q_0 a^t$ has growth for $a > 1$ and **exponential decay** 指数衰减 for $0 < a < 1$. State the time unit and whether time is continuous or restricted to whole periods. The basic exponential graph has y-intercept 1 and stays above the x-axis; $\log_b x$ has x-intercept 1 and vertical asymptote $x = 0$.

■ Worked example: repeated reductions multiply rather than subtract a fixed amount



equal time periods, changing absolute reductions

Known: an invented quantity starts at 100 units and decreases by 20% per whole period. The retained proportion is 0.8, so use a multiplicative model.

$$Q(n) = Q_0(1 - r)^n$$

$$Q(2) = 100(1 - 0.20)^2 = 64$$

The first decrease is 20 units and the second is 16. A fixed subtraction of 20 each period would be a different model.

2 Practice

2.1 Simplify $2^3 \times 2^{-5}$ to a single power and then to a fraction. [2]

2.2 Solve $\log_3 x = 4$ in the real numbers. [2]

2.3 Solve $5 \times 2^t = 40$. Sketch $y = 2^t$, labelling its y-intercept and horizontal asymptote. [3]

2.4 Solve $\ln(x - 1) + \ln 2 = \ln 10$. State the domain restriction. Sketch $y = \ln x$, labelling its x-intercept and vertical asymptote. [3]

3 Exam-style questions

3.1 An account starts with 500 units and grows by 4% at the end of each year, with no deposits, withdrawals or charges. (a) Write a model for the value after n whole years. (b) Find the value after 3 years to 2 decimal places. (c) Find the first whole year at which its value is greater than 600 units. [6]

[8]