

Solutions

2.1

Same-base products add exponents.

$$E = 2^{3+(-5)} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Correct power and fraction.

2.2

Convert the logarithmic statement to exponential form.

$$x = 3^4 = 81$$

Positive solution 81.

2.3

Divide both sides by 5.

$$2^t = \frac{40}{5} = 8 = 2^3$$

Thus $t = 3$. An increasing exponential curve through $(0, 1)$, approaching but never meeting $y = 0$ as t decreases, earns the final point. Label the y -intercept and asymptote.

2.4

The equation requires $x - 1 > 0$, hence $x > 1$. Combine logarithms of positive factors.

$$\ln(2(x - 1)) = \ln 10 \implies 2(x - 1) = 10$$

$$x = 1 + \frac{10}{2} = 6$$

Solution 6 satisfies $x > 1$, and $\ln 5 + \ln 2 = \ln 10$. For the separate sketch, draw an increasing logarithmic curve on $x > 0$ through $(1, 0)$ with vertical asymptote $x = 0$; label both features.

3.1

(a) Each year's multiplier is 1.04.

$$A(n) = 500(1.04)^n$$

Correct model.

(b) Substitute three completed periods.

$$A(3) = 500(1.04)^3 = 562.432 \approx 562.43$$

Correct rounded value.

(c) The strict threshold is $500(1.04)^n > 600$. Divide by 500 and use the increasing natural logarithm.

$$n > \frac{\ln(600/500)}{\ln 1.04} \approx 4.6486$$

Correct threshold. First whole year is 5: $A(4) \approx 584.93 < 600$ and $A(5) \approx 608.33 > 600$.

3.2

(a) $Q(0) = 200$ units. Retained proportion is 0.85, so the hourly decrease is 15%.

(b) Substitute four hours.

$$Q(4) = 200(0.85)^4 = 104.40125 \approx 104 \text{ units}$$

Correct 3 significant figures.

(c) Set the quantity equal to the target and take logarithms.

$$0.85^t = \frac{100}{200} = 0.5$$

$$t = \frac{\ln(100/200)}{\ln 0.85} \approx 4.27 \text{ hours}$$

Correct continuous time.

(d) First qualifying sample is at 5 hours because $Q(4) > 100$ and $Q(5) \approx 88.74 < 100$. For every finite t the positive base raised to t stays positive; the model approaches zero but never reaches it.