

4.4 Construct and trace algorithms with explicit input conditions

Name: _____ Class: _____ Date: _____

Total: 24 marks

KEY WORDS algorithm 算法 • flowchart 流程图 • pseudocode 伪代码

Objective

Construct and trace algorithms with explicit input conditions. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

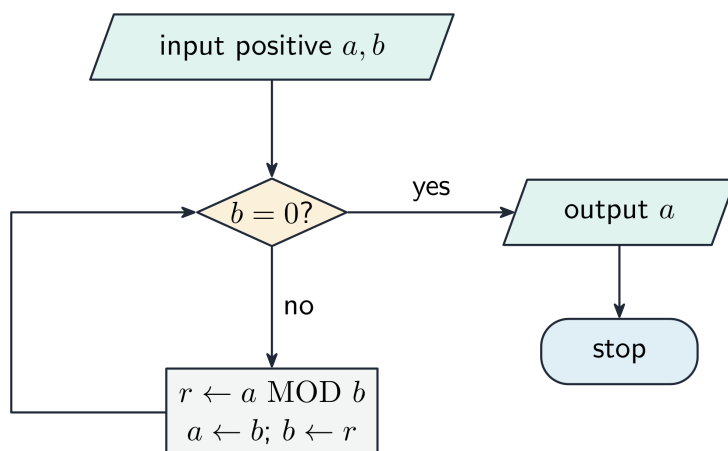
1 Key knowledge and worked examples

■ **Key knowledge:** a correct trace follows the stated step order and loop bounds

An **algorithm** 算法 states unambiguous steps for a task over defined allowed inputs. To solve the stated task it must produce the required result and terminate under those input conditions. A **flowchart** 流程图 uses labelled flow arrows, process boxes and decision branches; input/output uses a parallelogram.

Pseudocode 伪代码 describes the procedure without promising a particular language's exact syntax. On this sheet, assignment updates the left-hand variable, FOR bounds are inclusive, and MOD is the nonnegative remainder for nonnegative integers and a positive divisor. Trace variables after each stated update. Binary search requires an ordered list and consistent index/bound conventions; linear search can stop at an early match and does not always inspect every item.

■ **Worked example:** repeat a remainder step until the second number is zero



Known: start with positive integers a equal to 10 and b equal to 6. While b is nonzero, compute r as $a \text{ MOD } b$, then set a to b and b to r . Pairs after complete iterations are $(6,4)$, $(4,2)$, $(2,0)$, giving output 2. Temporary r preserves the remainder before a and b change. Each nonzero remainder is smaller than the previous positive b , supporting termination.

2 Practice

2.1 Start total at 0. For i equal to 1, 2 and 3, update total by adding i . List the total after each update and the final output. [2]

2.2 A linear minimum-finding procedure starts smallest at the first value of $[5, 1, 4]$, then compares each remaining value with smallest. List the two later smallest values and the final minimum. [2]

2.3 Draw a flowchart that inputs one real number n , outputs “nonnegative” if n is at least zero and “negative” otherwise, then stops. Label the decision branches. [3]

2.4 Binary search uses zero-based indices, inclusive low/high bounds and $\text{floor}((\text{low} + \text{high})/2)$. Trace its middle indices while finding 14 in the sorted list $[2, 5, 8, 11, 14, 17, 20]$, starting $\text{low} = 0$ and $\text{high} = 6$. [3]

3 Exam-style questions

3.1 A procedure is intended to add integers 1 through n for a nonnegative integer n . It starts $\text{total} = 0$ and $i = 1$, repeats while $i < n$, adds i to total , then increments i . (a) Trace it for $n = 4$. (b) Identify the boundary error and correct the condition. (c) State the correct outputs for $n = 4$ and $n = 0$. [6]

Solutions

2.1

Successive totals are 1, 3 and 6. Correct repeated-update method earns M1. Final output 6.

2.2

After comparing 1, smallest becomes 1; comparing 4 leaves it at 1. Correct update/retain sequence. Final minimum 1.

2.3

Input n followed by a decision testing $n \geq 0$. The yes branch outputs nonnegative and the no branch negative; both are labelled correctly. Both output branches reach a stop, with coherent arrows and appropriate process/decision/input-output symbols. Zero follows the nonnegative branch.

2.4

First middle is 3, value 11. Since 14 is greater, set low to 4; next middle is 5, value 17. Set high to 4; next middle is 4, value 14, found. The compared middle indices are 3, 5, 4.

3.1

- (a) The original loop adds 1, 2 and 3. It outputs 6 rather than 10.
- (b) The strict condition excludes n . Use i less than or equal to n .
- (c) Correct output for 4 is 10. For 0, the corrected loop executes zero times and outputs the initial total 0. The input restriction excludes negative and noninteger n .

3.2

- (a) First remainder is $48 \text{ MOD } 18 = 12$, giving pair (18,12). Next remainder $18 \text{ MOD } 12 = 6$ gives (12,6). Final remainder $12 \text{ MOD } 6 = 0$ gives (6,0), so output 6.
- (b) The remainder must use the old a and b ; overwriting them first changes the division. A temporary r preserves it.
- (c) For positive b , the new nonnegative remainder is strictly smaller than b . A strictly descending sequence of nonnegative integers reaches zero, so the procedure terminates. Euclidean remainder replacement preserves common divisors, and the final output is the greatest common divisor.
- (d) The zero test immediately outputs a without taking MOD by zero.