

4.2 Convert positional numbers and calculate in binary

Name: _____ Class: _____ Date: _____ Total: 24 marks

KEY WORDS number base 进制 • binary 二进制 • hexadecimal 十六进制

Objective

Convert positional numbers and calculate in binary. Original GAC004-aligned practice, not an official assessment. Show the method as well as the result. All applied scenarios are invented classroom problems.

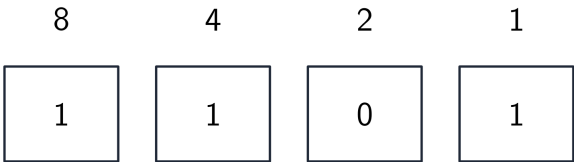
1 Key knowledge and worked examples

■ Key knowledge: a numeral must be read in its stated base

A positional **number base** 进制 b uses digits from zero to b minus one and place weights b^0, b^1, b^2 and so on. **Binary** 二进制 uses digits 0 and 1; **hexadecimal** 十六进制 uses 0 to 9 and A to F. Four bits correspond to one hexadecimal digit, including leading zeros where width matters.

Binary addition carries at 2. State whether a result is an unrestricted mathematical numeral or must fit a fixed-width representation. An unsigned n -bit pattern represents values from zero to $2^n - 1$. A carry beyond a fixed width may indicate overflow; it must not be silently retained or discarded without stating the convention.

■ Worked example: place weights determine the decimal value



place weights above binary digits

Known numeral 1101₂. Use weights from right to left: 1, 2, 4 and 8.

$$V = \sum d_i 2^i$$

$$V = 1(8) + 1(4) + 0(2) + 1(1) = 13$$

The same value is D in hexadecimal. Leading zeros would not change this nonnegative value but can record an intended width.

2 Practice

2.1 Convert 10110₂ to decimal. [2]

2.2 Write decimal 13 as an unsigned four-bit binary pattern. [2]

2.3 Convert 2F₁₆ to decimal and to an eight-bit binary pattern. [3]

2.4 Calculate $1011_2 + 0110_2$ as an unrestricted binary numeral. Then state whether it fits in four unsigned bits. [3]

3 Exam-style questions

3.1 (a) Write decimal 45 as an unsigned eight-bit pattern and hexadecimal numeral. (b) Subtract 00101101_2 from 01000000_2 as nonnegative integers. Give the result as eight bits and decimal. [6]

3.2 An unsigned eight-bit counter starts at 11111010_2 . A calculation adds decimal 10. (a) Find the initial decimal value and the unrestricted sum. (b) Write the unrestricted sum in binary. (c) Decide whether it fits the counter. (d) Under a separately specified wraparound rule that keeps only the lowest eight bits, find the stored pattern and decimal value. Explain why the stored value differs from the mathematical sum. [8]

Solutions

2.1

Use binary place values.

$$V = 1(16) + 0(8) + 1(4) + 1(2) + 0(1) = 22$$

Decimal 22.

2.2

Decompose 13 into weights 8, 4 and 1. Four-bit result 1101.

2.3

F represents 15.

$$V = 2(16) + 15 = 47$$

Decimal 47. Each hexadecimal digit supplies four bits, so binary pattern 00101111.

2.4

Column addition gives binary 10001. Carry method earns M1 and correct result A1. It represents 17 and exceeds the four-bit unsigned maximum 15, so it does not fit.

3.1

(a) Decompose 45 as $32 + 8 + 4 + 1$. Eight-bit pattern 00101101. Hexadecimal 2D.

(b) Subtract values or use binary borrowing.

$$V = 64 - 45 = 19$$

Decimal result 19. Eight-bit pattern 00010011.

3.2

(a) Initial value is $V = 128 + 64 + 32 + 16 + 8 + 2 = 250$. Mathematical sum $S = 250 + 10 = 260$.

(b) Use place weights to represent 260. Binary numeral 100000100.

(c) Eight unsigned bits can hold at most $2^8 - 1 = 255$, so overflow occurs.

(d) Keep the low eight bits under the explicitly stated rule. Stored pattern 00000100, value 4. The high 256-weight bit is discarded, so this stored value is the remainder modulo 256 rather than the mathematical sum 260.